Fall 2015

Dynamic Project Time and Cost Evaluation Under the Combined Influence of Contract, Quality, and Uncontrollable Change

Thomas V. Hammock
Old Dominion University, thamm006@odu.edu

DYNAMIC PROJECT TIME AND COST EVALUATION UNDER THE COMBINED INFLUENCE OF CONTRACT, QUALITY, AND UNCONTROLLABLE CHANGE

by

Thomas V. Hammock
B.S. May 1987, Tennessee Technological University
M.S. May 1989, Virginia Polytechnic Institute and State University
M.B.A. December 1992, University of Richmond

A Dissertation Submitted to the Faculty of
Old Dominion University in Partial Fulfillment of the
Requirements for the Degree of

DOCTOR OF PHILOSOPHY

MECHANICAL ENGINEERING

OLD DOMINION UNIVERSITY
December 2015

Approved By:

Han P. Bao, (Director)

Gene J. -W. Hou, (Member)

Duc T. Nguyen, (Member)

ABSTRACT

DYNAMIC PROJECT TIME AND COST EVALUATION UNDER THE COMBINED INFLUENCE OF CONTRACT, QUALITY, AND UNCONTROLLABLE CHANGE

Thomas V. Hammock
Old Dominion University, 2015
Director: Dr. Han P. Bao

This dissertation proposes a method that quantifies the dynamics caused by disruptions due to multiple changes that can occur during the progression of a large scale project. To begin, equations consisting of multiple parallel event sequences are derived that define the relationship between individual process events and the final project completion time. Matrix notation and a precedence matrix concept are used in this derivation to produce equations that are best suited for computer programming applications. This relationship is the foundation upon which the equations are built to calculate the variation impact on each event of a large scale project. Pursuing this further, the Analytical Hierarchy Process is used to rank Contract, Quality and Uncontrollable Variation types to generate two multiplication factors. One factor accounts for the variation type, and the other captures the variation influence relative to each event. These factors are used to define and formulate the equations used to quantify the impact of variation on each project event. Matrix notation, binomial variables, and Hadamard operators are used in the formula to optimize the result's applicability to large data sets required for large scale projects. In the final analysis, the impact equations are used in a Monte Carlo simulation to produce each impact probability. These impact probability results are helpful tools for managing cost and evaluating design options. The Monte Carlo results are also used to reinforce the proposed process by quantifying the impact of change and providing a measurable metric for performance accountability. Examples and illustrations are used in each derivation to better convey the proposed concepts and application

This dissertation is dedicated to my mother and father.

ACKNOWLEDGEMENTS

The successful completion of this dissertation is largely a result of the contributions of many people. I thank my committee members for their guidance and care in editing of this manuscript. Also, Dr. Han Bao deserves special recognition for his untiring efforts and help.

NOMENCLATURE

F = the effective variation type weighting

T = the ranking matrix for the type of variation involved

R_k = ranking matrix for the P events due to the presence of the kth variation

μ_k = the binary variable associated with the variation type

ρ_k= the binary variable associated with the events

K = the types of variations

k = the variable used to represent variation type, k= k^{th} variation type, k=1 to K

Δ = the change in time, or extraneous time due to variation, i.e. Δ_p^k is the change in time for the k variation type during the event p

P = the number of events

p = variable used to represent event. p= p^{th} event, 1 to P

$[e]_0$ = the initial event duration matrix

$[e]_n$ = the event duration matrix populated with the "new" events durations calculated for variation n

[E] = the event completion matrix

$[E]_n$ = the event completion matrix for variation n

[CM] = the cumulative matrix

N = the total variations during project progression, for our illustration N=4

n = the variable used to represent the nth variation, n =1 to N, i.e. for variation one, or V_1, n=1

t_i = time of variation impact i

e_i = time duration of event i

M_k = the effective event ranking conversion

NOMENCLATURE/ACRONYMS (cont.)

Δ_n = change in task times due to variation n

s = event variation impact column matrix

ϑ = the time during project progression that variation occurred

TABLE OF CONTENTS

Chapter Page

LIST OF TABLES

LIST OF FIGURES

Figure Page

CHAPTER 1

INTRODUCTION

1.1 Purpose

The purpose of this dissertation is to propose and illustrate a model for evaluating the impact of variation to large scale projects. The research in this dissertation applies to modeling and simulation of cost estimation models as a field of study, and the proposed model is specifically for large scale projects. Calculating impact due to variation is a critical element in planning, production, and business processes in large scale construction projects. The model is also a critical element in information systems and risk management. The topic for this dissertation is dynamic project time and cost evaluation under combined influence of contract, quality, and uncontrollable change. The proposed model for variation impact uses the Analytical Hierarchy Process to rank different classifications of variation and the relative ranking of project events. It also uses Monte Carlo simulation to estimate the probability of different impacts on projects. This dissertation further explains the proposed mathematical model with illustrations to show how to apply the model.

1.2 Problem Statement

Predicting the dynamic project time and cost evaluation under combined influence of contract, quality, and uncontrollable change continues to challenge areas of large scale projects such as accounting, finance, planning, and production. Specifically, the existing method of accounting for project variation to be used for cost control management is inaccurate and lacking. This study explores the relationship between the types of change during project progression and explains a method to predict the impact of variation in a large scale project.

1.3 Large Scale Projects

A project can require months or years to complete and cost millions, even billions, of dollars. This is particularly true in large scale construction projects such as bridges, tunnels, buildings, ships, and airplanes. Managing these projects consists of a complex and integrated array of decisions, actions, and communications necessary to complete projects successfully. General project planning practices are reviewed here to help explain the formulas developed in chapter 3. A key element to successful project management is to document what the client wants by identifying all the work to be done and the personnel available to do the work. This communication will also recognize all the risks involved with doing the work, anticipate any problems that could interrupt the project once it has started and calculate and publish project time and cost targets. After gathering all the necessary information to define and scope the project, a baseline schedule and baseline budget is developed. The cost of the project is estimated and documented in the project proposal. When the decision is made to go forward with the project, it is important to have a plan to monitor and control costs. This monitoring is set at intervals to allow necessary intervention to avoid cost overruns. The cause of the actual cost exceeding budget in most projects is poor cost control. Other factors that contribute to project cost overruns are poor cost estimates and a lack of standards or rules for developing estimates and cost control processes.

The project does not generate revenue during the scoping, planning, engineering, and construction phase, and many times dependent cost elements consume a majority of the project funding. Some of these time dependent elements are labor, engineering and leased equipment. These time dependent costs consume a large amount of cash and are a major cost concern.

1.4 Variation

There are many reasons why a project will deviate from the original contract. These changes are often called change order or variation order. Some changes are initiated by the owner; other changes are due to technological advances or design oversight. When changes are introduced prior, during or after construction work has been executed, the impact of these changes can drastically affect project performance. Design changes to the product during the manufacturing or construction process, as experience proves, are inevitable but can be properly managed and controlled. These changes usually directly affect the contract in terms of time and cost. The owner and contractor both must be knowledgeable as to how to control and protect their interests. Setting and following the control systems in place is imperative in order to successfully manage changes and design variations. It is important to incorporate the interdependence of design and construction in the planning phase of a large scale project. The design and construction processes are coupled activities. Design is a process of drafting and calculating the specifications for a large scale project, usually documented using detailed plans and drawings; construction is a process of allocating labor and resources necessary to turn the design into the desired product. Construction is the process of building the design calculated and conceptualized by the architects and engineers. In both construction and design, various operational tasks must be accomplished with a variety of priorities and other dependencies among the different tasks.

For a large scale project, the proposed and actual costs to complete a project are usually considerably different. This difference is referred to as variation. The relationship between large scale project design and variation must be defined in order to take advantage of opportunities for controlling the impact of variation. Identifying and quantifying the relationships between design

changes and the necessary events to produce a product in order to mitigate or avoid variation impact is vitally important to large scale projects. The customer's goal is to have the changes incorporated into the original product without increasing cost or delivery time and without compromising the performance of the other systems in the product.

Consider, for example, a large scale construction project such as a bridge.

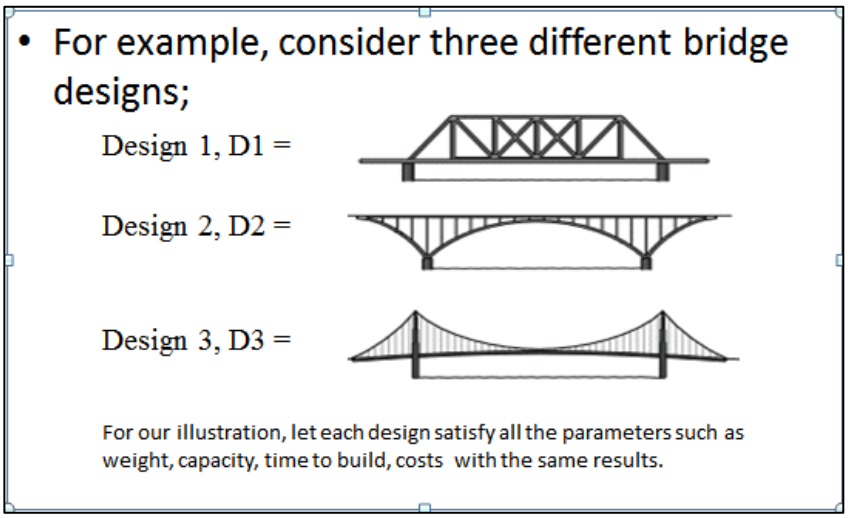


Figure 1.1: Three Different Designs

In figure 1.1, the design of each bridge is conceptualized as a function of the contract parameters and impact that identical variations have on each design. The independent variables are the contract parameters and variations during project progression. The dependent variable is the design of the bridge. For our illustration, the parameters are held constant by letting each design satisfy all the contract parameters, such as capacity, costs, and time to build, with the same results. Those parameters are set as follows.

- D1=capacity of 40 tons, costs 2 million, will take 10 weeks to build.
- D2=capacity of 40 tons, costs 2 million, will take 10 weeks to build.

- D3=capacity of 40 tons, costs 2 million, will take 10 weeks to build.

Define activities: For this bridge illustration, four activities will be used: scoping, engineering, ordering, and fabricating the bridge.

- Scope = 3 weeks. In the **scope** portion of the design process the customer request is defined**.** After the request is defined the requirements needed to make the design change are calculated. Knowing the requirements needed for the design change, each event of the project is examined to determine how the design change impacts the event.
- Engineering = 2 weeks. During the **engineering** part of the design**,** engineering calculates the necessary dimensional or component modifications needed to bring the concept to reality. Drawing changes are made and issued to the necessary departments for processing.
- Ordering = 1 week. Obtaining the material and components required for the design change is performed during the **order** portion of the process. This process includes notifying contractors, changing and issuing work orders and work packages and canceling materials not needed.
- Fabricating = 4 weeks. In the **fabrication** portion the material is fabricated and processed, components are installed and the system is tested.

Figure 1.2 below shows the Gantt Chart for building the bridges.

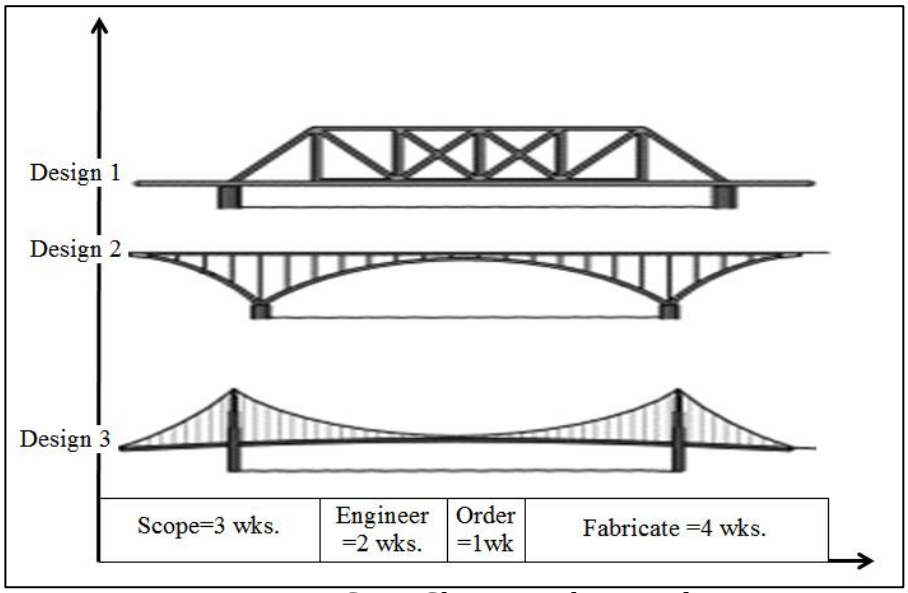


Figure 1.2. Gantt Chart For Three Bridges

Next, there will be four variations introduced that occur during the project progression to illustrate the impact that each variation has on the project. For example, after the first week of the project the customer approaches the supplier requesting that the bridge be 2 feet wider than the original design (see Figure 1.3). Note that 1 week after beginning the project that is the scoping process and no engineering, ordering, or fabricating has been performed. This request being made at the beginning of the project progression minimizes the impact. If this request were made in the final week of project completion then it would impact all phases of the project. In general, the earlier the variation, the smaller the impact it has on the project.

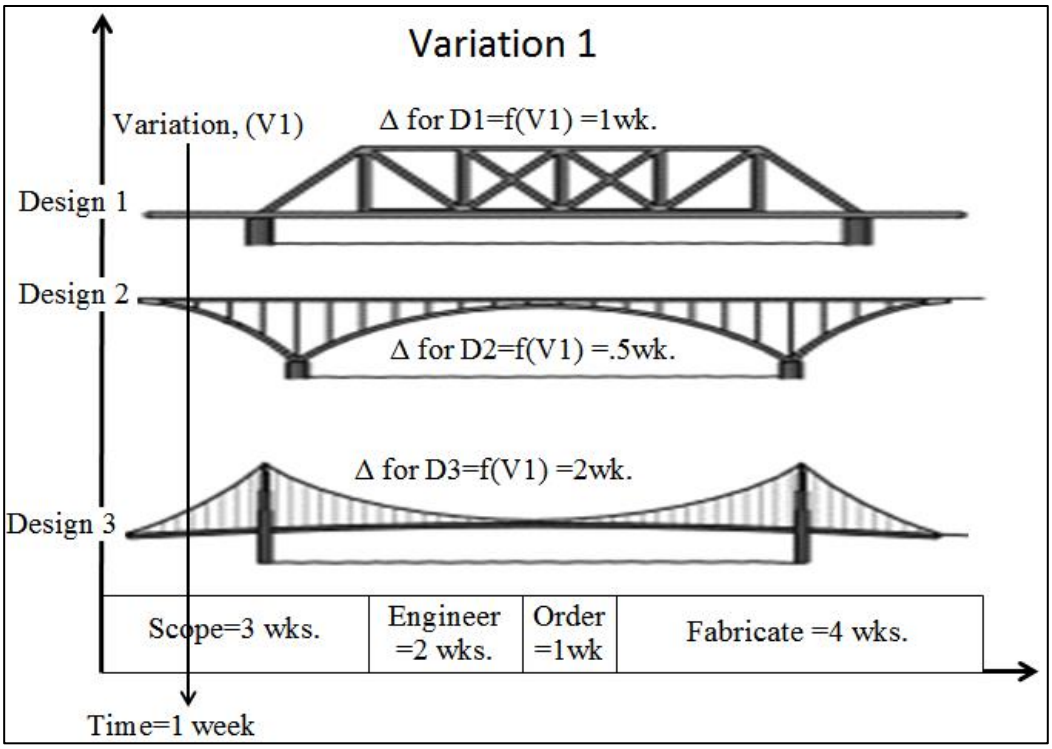


Figure 1.3. Bridge Variation #1

Note that increasing the bridge width has different impacts depending on the design of the bridge. The design 1 (D1) bridge will require 1 week for adding 2 feet to the width; the design 2 (D2) bridge requires .5 week; the design 3 (D3) bridge requires 2 weeks. Why the difference? It may be that one design requires more engineering calculations, more labor for fabrication, or a longer lead time for materials. The concept illustrated here is that different designs will have different impacts from the same variation.

For the second variation shown in figure 1.4, the customer approaches the supplier two weeks after the project has been started and requests that the capacity be increased from 40 tons to 50 tons. The later into the project that the change requests are made, the more impact they will have on the project.

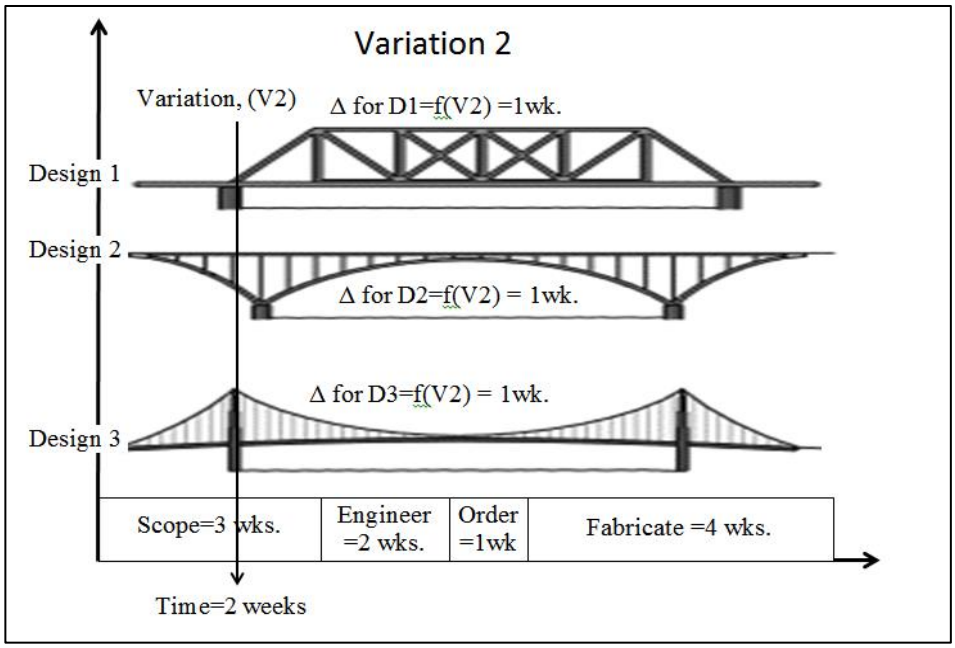


Figure 1.4. Bridge Variation #2

For this particular request, the impact is the same for all three designs. It will take 1 week extra to modify the original designs for an increased capacity from 40 to 50 tons.

The third variation shown in figure 1.5 is a quality variation that occurs at week 4 in that the fasteners do not meet required specifications. This particular problem causes a 2 week delay for the design 1 bridge, a 1.5 week delay for the design 2 bridge and a 3 week delay for the design 3 bridge.

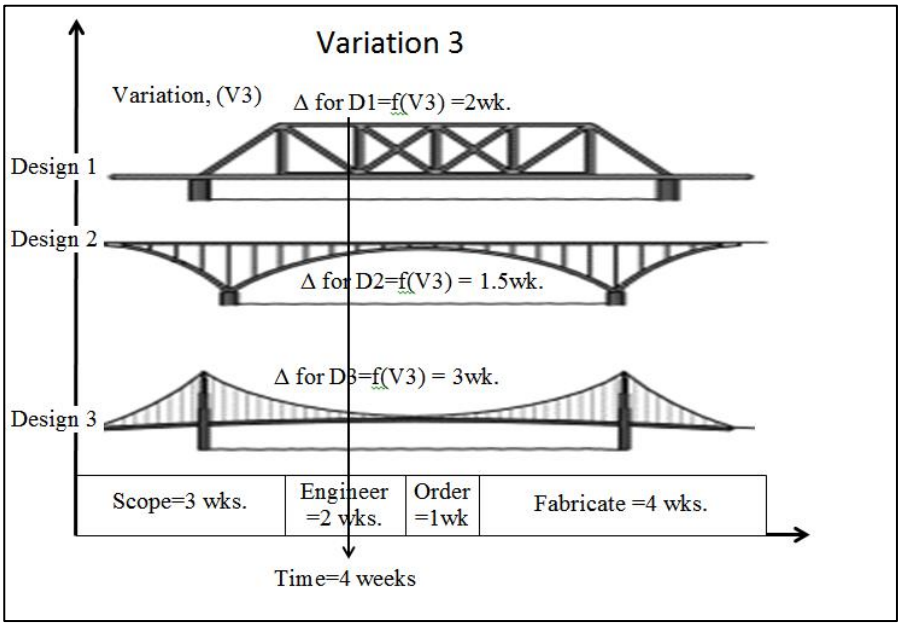


Figure 1.5. Bridge Variation #3

The fourth variation 7 weeks after the project has started is in the form of a city citation requiring that the handrails be modified to meet code. This delayed the design 1 bridge by 4 weeks (see figure 1.6), the design 2 bridge by 2 weeks, and the design 3 bridge by 5 weeks.

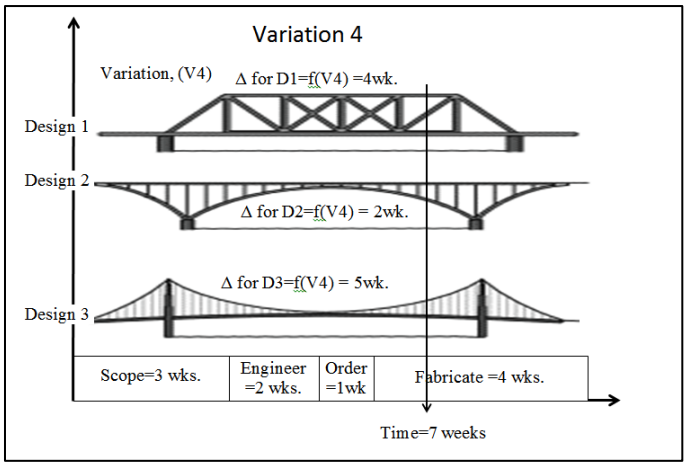


Figure 1.6. Variation 4 at week 7, Handrail Requirement

When all four variations are superimposed on one chart in figure 1.7, the total delays are: 8 weeks for the design 1 bridge, 5 weeks for the design 2 bridge, and 11 weeks for the design 3 bridge.

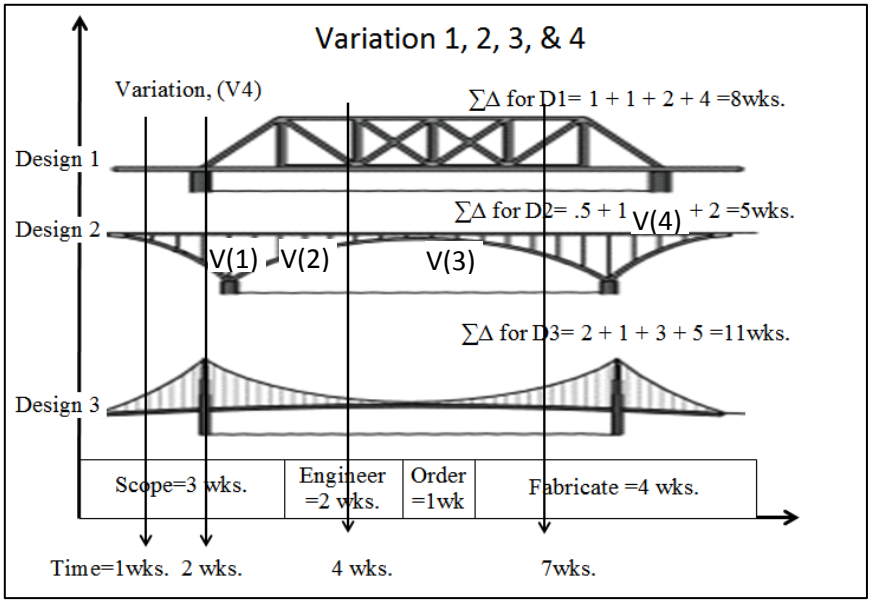


Figure 1.7. Four Variations

Customer change requests and code requirements are traditionally the responsibility of the technical sales team. The quality department usually resolves problems such as the defective fasteners in the illustration above, but if the relationship between design and variation was defined, the contractor and customer would be on common ground when a design is selected by knowing the impact of variation during project progression.

In our bridge illustration, the impact that variation had on the project was given. The bridges can be compared for a number of variations. In our illustration, the number of variations, N, is four, i.e. N =4. Using the initial completion time as a base (10 weeks), a variation impact factor can be calculated for each design by dividing the completion time after variation impact by the initial

completion time. For D1, (18/10) =1.8, D2=1.5, and D3=2.1. In general, if the relationship between design and variation is defined, variation impact can be evaluated.

1.5 Defining the Dependent and Independent Variables

This dissertation derives the relationship between design and variation and quantifies the impact that variation has on project time and cost at completion. It proposes methods for solving what the variation is, when the variation occurs, and where the variation impacts. Thus, the dependent and independent variables are defined as follows.

Given: the project plan, the design, the sequence of events, the rework factors

Independent variables: the number of variations, the time that the variation occurs, the type of variation

Dependent variables: variation impact

1.6 Research Applications

Defining the impact of variation on a specific design before the project begins helps to:

- Mitigate contract litigation
- Bound spending
- Optimize design
- Quantify cost to support design research
- Control costs
- Justify funding from specific industries for defining their Precedence Matrix, rework factor, and variation initiation profile

CHAPTER 2

LITERATURE REVIEW

2.1 Existing Classification of Delay

There is an abundance of literature discussing the impact of change on a project. A few examples include: (Brown, D. W. 1988), (Construction Industry Institute. 1988), (Hester, W. T. et al. 1988), (Leonard, C. A. 1987). Change has also been cause for litigation between the customer and supplier because of missed deadlines and increasing costs (Building Research Advisory Board 1978). Living in a world where the international market borders are melting is increasing competition in the area of efficiency and flexibility. The many sightings show that this is an ongoing occurrence (Brown, D. W. 1988), (Construction Industry Institute. 1988), (Hester, W. T. et al. 1988), (Leonard, C. A. 1987), (Salvahan, 2011). There are several helpful suggestions for better managing design change, but there remains an abundant opportunity for developing methods to improve the change process (Dellon, A. L. et al. 1988), (Hester, W. T. et al. 1988). It is also important to note that the challenge of budgeting and forecasting the cost of large projects is largely due to implementing design changes that occur after the project has started. These design changes can be from initial contract proposal oversights or from implementing technological advances that occur during the duration of the build process. The cost of these changes increase exponentially (El-Haik & Al-Aomar, 2006) p 134 and 135,(Mather & Management, 1999) p 200 and 201 as the project advances.

2.2 Existing Methodologies Used to Evaluate Project Progression

Many references use the term "delay" instead of variation to describe an event that changes the original planned timeline of a large scale project. One common practice of classifying delay is by using three categories: critical, excusable, and compensable (Ndekugri et al. 2008). The first category, critical or non-critical delay, applies to events that impact the original timeline's critical or non-critical completion path. A delay is classified into one of two possibilities in the second category, excusable or non-excusable, depending on whether the supplier or contractor is excused for the extension of time because of the delay. A delay is compensable or non-compensable depending on whether or not the customer is going to pay for the cost consequences caused by the delay. (Kumuraswamy et al. 2003; Ndekugri et al. 2008).

One goal of all Delay Analysis Methods is to show how project delays affect various activities and, ultimately, the project completion date. Another goal of the Delay Analysis Method is to allocate each part of the overall project delay to the responsible party involved in the project. (Ndekugri et al.2008; Stumpf, 2000). This allocation serves as the basis for cost and/or time settlements for the contracting businesses. The general business practice for contractors to collect costs due to delays is for them to show that the delayed events, according to terms of the contract, were at the risk of the owner, and prove the delay moved the project completion date. The critical path method (CPM) of planning can be used to prove the delay moved the project completion date. That is why Delay Analysis Methods use a critical path method to justify time-related claims such as delay of project completion time and increasing cost (Wickwire et al. 1989; Bramble and Callahan, 2000).

Some common methodologies such as As-planned vs. As-built, Impacted As-planned, Collapsed As-built, Window Analysis, and Time Impact Analysis and are reviewed to give an

overview of the current conventions used for analyzing delay or variation. Details of each method are beyond the scope of this dissertation. The references cited in each review can be used for further information.

The *As-planned vs. As-built* methodology contrasts the events of the original CPM baseline plan with those of the as-built plan to assess and evaluate the delays. This methodology has advantages in that it is inexpensive, simple and easy to understand (Ndekugri et al. 2008). Two shortcomings of this method are its inability to account for changes in the critical path and its difficulty in dealing with complex delay situations (Lucas, 2002; Lovejoy, 2004; Pickavance, 2005).

The *Impacted As-planned* methodology is where project delays are included in the CPM as planned events to show how the delay changes the project completion date. The change between the original planned completion date and the modified planned completion date is the amount of time caused by the delay. One advantage of this methodology is that it does not use an as built schedule to calculate the amount of time caused by each delay. Some weaknesses of this method are that it fails to account for any changes in the critical path and sequence of events. (Ndekugri et al. 2008; Pickavance, 2005; Wickwire and Groff, 2004).

The *Collapsed As-built* methodology uses an as-built CPM schedule with all the project delays incorporated. This schedule is 'collapsed' by taking away the delays that occurred during project progression. This collapsed schedule shows how the project would have been completed without the delays. One strong point of this method is its level of accuracy; however, it fails to deal with changes in the critical path and work needed in finding the as-built critical path. (Lovejoy, 2004); Wickwire and Groff, 2004).

The *Window Analysis* method utilizes the as-built CPM schedule by dividing it into sections or 'windows' for analysis. Major project milestones, such as changes in the critical path, major delays, or completion of major project components, define the boundaries of the window. These milestone events determine the quantity and time interval of the windows for the total project. This method of analysis gives more accurate results as the number of windows increase. (Hegazy and Zhang, 2005; Ndekugri et al. 2008). Window analysis is performed by updating the plan within the first time interval by using as-built data and incorporating all the delays occurring in that time interval. The remaining as-planned schedule beyond this interval is held unchanged. The change in the original plan and the as-built plan that includes all the delays is the amount of project in the time interval. This process is repeated for each time interval to calculate the impact of each delay and total project delay. This method accounts for the changing nature of the critical path, but it is expensive because of the large amount of work needed to perform the analysis. (Bordoli and Baldwin, 1998; Finke, 1999; Galloway and Nielsen, 1990)

The *Time Impact Analysis* methodology is similar to the window method described above, but instead of examining time intervals between milestones, the analysis focuses on a specific delay. The Time Impact Analysis method considers the impact of delays chronologically. The actual project status before and after the delay is used to quantify the delay in the project completion date. This is one of the most reliable techniques, but it is expensive because of the large amount of work required to perform the analysis. (Alkass et al., 1996; Pickavance, 2005).

2.3 A Review of Work Done on Process and Project Simulation

Sterman (2000) discusses how project budget models have been used to fight claims and to forecast the financial consequences of various decisions. These models are confidential and

are developed for a specific company. There is not a model proposed available to the public that gives the impact of variations as a relationship of design to the number of variations. Sterman does not present a general format that can be tailored to fit different types of industry by structuring the precedence matrix and AHP to fit a specific large scale process.

System dynamics in modeling is where a dynamic hypothesis is a working theory of how the variation happened. This hypothesis focuses the customer's and supplier's efforts on structures that model reality. System dynamic modeling methods "are not able to easily accommodate multiple and/or concurrent disruptions caused by different project parties"(Nelson, Derek, 2011). The methods presented in this dissertation accommodate multiple variations, with multiple types of variation, and account for their dependence by using a precedence matrix.

CHAPTER 3

DERIVATION OF THEORETICAL FOUNDATION

3.1 The Relationship between Project Completion Time and Individual Events

As stated in chapter 2, identifying and quantifying the relationships between design changes and the necessary events to produce a product in order to mitigate or avoid their impact is vitally important to large scale projects. The customer's goal is to have the changes incorporated into the original product without increasing cost or delivery time and without compromising the performance of the other systems in the product. In order for the supplier or contractor to communicate the disruption caused by a change, it is important that the details of the original contract timeline agreement be established and documented. The problem is that each change situation is different, and many times there are no standards for implementing or tracking the change. Another problem is that there are no procedures in place to account for the difference between the original and actual project timeline or the initial and actual change timeline performance. This dissertation proposes using matrix notation to represent the original project timeline. Matrix notation is defined as the precedence, the event, the cumulative, the final timeline matrices. Matrix notation gives visibility to the event details required to accomplish the tasks necessary for project completion and is an efficient way to handle the volume of data inherent to large scale projects. Using matrix notation also has the benefit of allowing us to use matrix arithmetic and matrix algebra to calculate the impact that design change has on the original project timeline matrix. For a large scale project that is in progress, there are two critical issues that must be resolved before we can apply lean manufacturing methods to the design change process. These issues are:

1. What events in the project does the change impact or affect, and

2. How the change in the event will affect the project.

To answer the second issue, one must know if the change impacts constraining events. In other words, if the change impacts non-constraining events, then the change in that event might not influence the finishing time of the final product. This is important because it allows us to separate and quantify original project plan variances from initial change impact variances. The contractor is usually responsible for such changes due to quality discrepancies. The customer is responsible for the change impact on the original project forecast such as contract changes. Separating and quantifying these responsibilities is essential for managing the project successfully. Below, an illustration is used to develop a general equation for the original project timeline and individual project events. This general equation is further developed to express the variance in both the original project timeline and the impact that a change has on the project.

<u>Derivation, using the bridge illustration</u>

- Let ei = the time duration for each activity i.
- e1 = scoping the job = 3 weeks
- e2 = engineering the job = 2 weeks
- e3 = sourcing and ordering material = 1 week
- e4 = Fabricating or building = 4 weeks

A Gantt chart below, shows each activity on a timeline;

e1=3wks	e2=2wks	e3=1wk	e4=4wks

I----------→ Time in weeks

Figure 3.1 Sequential Events

Figure 3.1 is a Gantt chart illustrating a series of sequential events for constructing a bridge. Column [E] is used to represent the time for each event to be completed. The cumulative time that preceded each event is represented by the column matrix [CM]. For example, the cumulative time that preceded e_2 was the 3 weeks of e_1. In matrix form,

$$\begin{bmatrix} 3 \\ 2 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 6 \\ 10 \end{bmatrix}. \qquad \text{Eq. 3.1a}$$

3.2 The Precedence Matrix

In equation 3.1, experienced planners or project managers that are qualified to accurately estimate the time needed to perform a certain task provide the values for the event matrix [e]. The matrix [CM] is used to develop the mathematical relationship between individual events and the final project timeline. The values for [CM] are calculated utilizing a tool called the precedence matrix. The precedence matrix for the sequence of events shown in figure 3.1 is defined and illustrated in figure 3.2.

Event j

$$[PM] = \text{Event i} \quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Figure 3.2. The Precedence Matrix for the Bridge Illustration

Where PM_{ij} indicates the presence (1) or absence (0) of a precedence between event i and event j. For example, $PM_{32} = 1$ means that event e_3 (sourcing and ordering materials) follows event e_2 (fabricating). $PM_{31} = 0$ means that event e_3 does not follow e_1.

As illustrated in the examples to follow, projects that have events sequences flowing from left to right and parallel sequences of events flowing from top to bottom will have constraints between events located in the lower triangle of the precedence matrix. The cumulative matrix [CM] for projects with events flowing from left to right and sequences flowing from top to bottom is obtained through the following procedure:

Procedure

For i = 1,

$CM_1 = 0$, $E_1 = e_1 + CM_1 = e_1$

For i = 2: P (P is the total number of events)

$CM_i = \text{Max}\ [(\text{ if } (PM)_{ij} = 1, E_j, 0) \text{ for } j = 1 \text{ to } i\text{-}1]$

$E_{i+1} = e_{i+1} + CM_{i+1}$

end

Note, the procedure for obtaining [CM] for a project that has event constraints located in the upper precedence matrix triangle is covered in Appendix D.

In equation form,

$$[CM] = \begin{cases} 0 \text{ if } i = 1 \\ \text{Max} \left\{ \text{IF}(PMij = 1, Ej, 0) \right\}_{j = 1,2,3\ldots i-1} \text{ if } i \neq 1 \end{cases} \qquad \text{Eq. 3.2}$$

3.3 Utilizing the Precedence Matrix on a Sequence of Events

- The CM3 element, i=3, PM32=1, E2=5, all other PM in row 3 are 0, so Max is 5, CM3 ←5

- $E^T = [\,3 \quad 5 \quad 6 \quad 10\,]$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 6 \\ 10 \end{pmatrix}$$

- The CM4 element, i=4, PM43=1, E3=6, all other PM in row 4 are 0, so Max is 6, CM4 ←6

- $E^T = [\,3 \quad 5 \quad 6 \quad 10\,]$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 6 \\ 10 \end{pmatrix}$$

- The first element, I = 1, so CM1←0,
- The CM2 element, i=2, PM21=1, E1=3, all other PM in row 2 are 0, so Max is 3, CM2 ←3

- $E^T = [\,3 \quad 5 \quad 6 \quad 10\,]$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 6 \\ 10 \end{pmatrix}$$

3.4 Utilizing the Precedence Matrix on Two Parallel Sequence of Events

Next, consider installing a pump as shown in figure 3.3 using the sequence of events shown in figure 3.4.

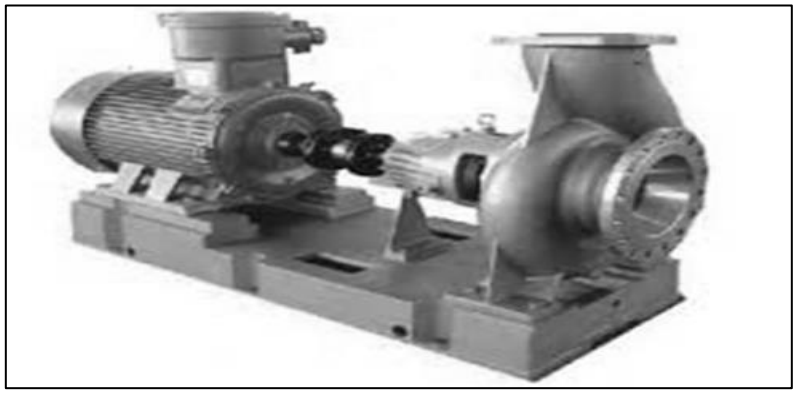


Figure 3.3. Installing a Pump

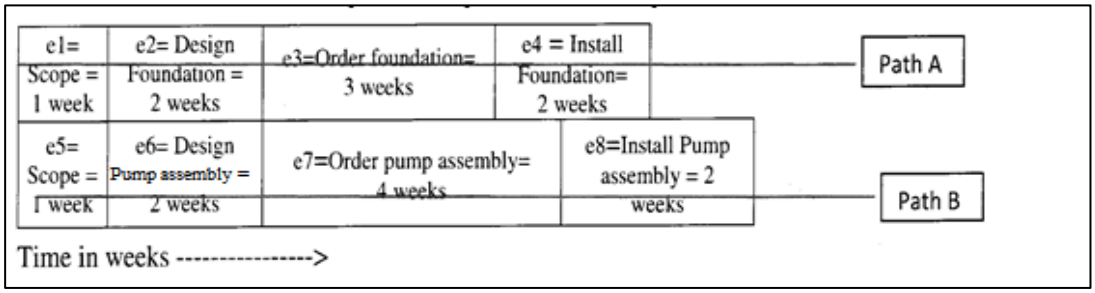


Figure 3.4. Two Parallel Uncoupled Sequential Events

The precedence matrix for these two uncoupled sequence of events is

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 3 \\ 6 \\ 0 \\ 1 \\ 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \\ 1 \\ 3 \\ 7 \\ 9 \end{bmatrix} \quad \text{. Eq. 3.3}$$

The precedence matrix shown above requires e1 to be done before e2, e2 to be done before e3, e3 to be done before e4, etc.

Next, consider the two parallel sequential events that are coupled. The coupling is due to the constraint of requiring the foundation to be finished before the pump assembly can be installed. The two critical paths are shown below in figure 3.5.

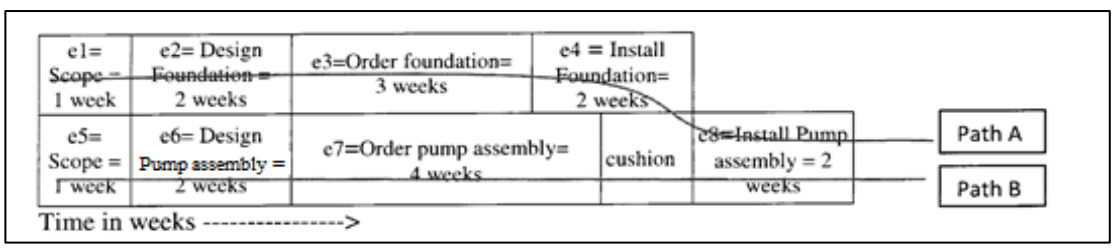


Figure 3.5. Two Parallel Coupled Sequential Events

The precedence matrix for these two coupled sequence of events is

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 3 \\ 6 \\ 0 \\ 1 \\ 3 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \\ 1 \\ 3 \\ 7 \\ 10 \end{bmatrix} \quad \text{Eq. 3.6}$$

3.5 The Rework Matrix

To solve for the impact that variation has on a project, the rework that the variation causes must be calculated. To help quantify the rework caused by variation, the variations will be separated into their major types for analysis.

$$\Delta_n = \begin{bmatrix} \Delta t_1 \\ \Delta t_2 \\ \Delta t_3 \\ \Delta t_i \end{bmatrix}_n \qquad \text{Eq. 3.5}$$

Equation 3.5 expresses variation n, such as variation 1: design change at week 1. For our illustration, variations are classified into K=3 types of variations. Contract is k=1, quality is k=2 and uncontrollable is k=3 variations. Each type of variation is ranked relative to one another in order to associate the amount of rework required due to each variation expressed as a multiple of the initial time needed to complete each event.

If all classes aren't included in the variation, then a convention is used to maintain the correct magnitude for rework. The total variation is the sum of each variation expressed.

$$\text{Total } \Delta = \sum_{n=1}^{N} \Delta_n \qquad \text{Eq. 3.6}$$

The time increase due to each variation is represented with the symbol Δ_p^k where k is the variation type, and p is the event during which the variation occurs. For our bridge construction example, the sequence of events and the variations occurring during project progression are illustrated and denoted using the variables shown in figure 3.6 below.

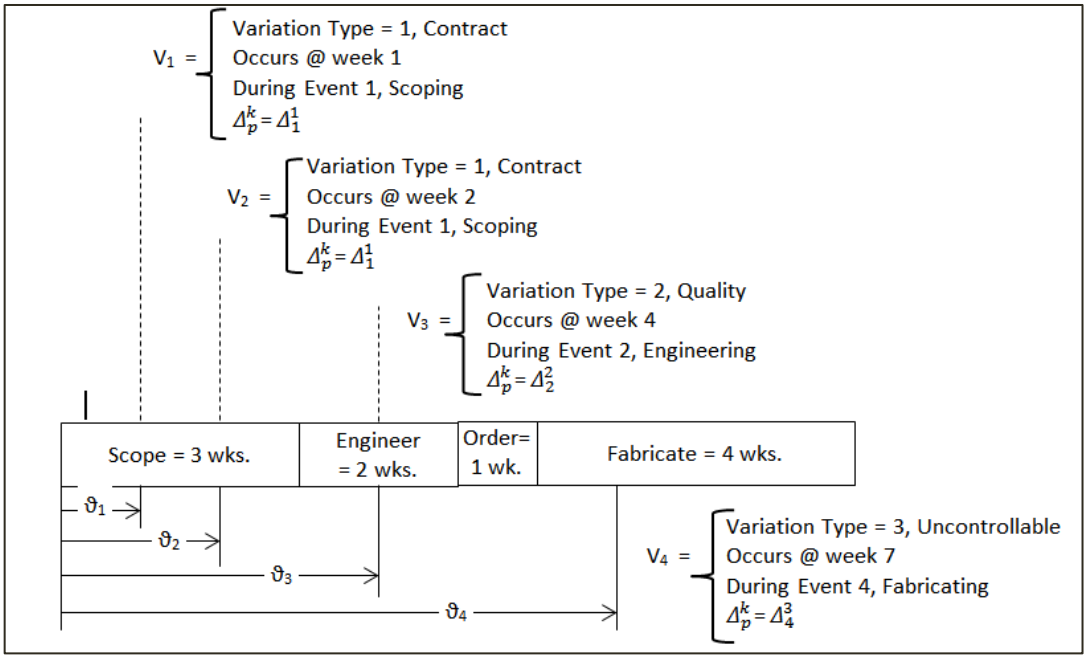


Fig. 3.6 Variation Impact

In the bridge illustration, V_1 occurs during the scoping event (p= 1) of the project, 1 week after the project started. V_1 is a contract type of variation, k=1. V_1 will impact all the activities that occurred during the first week (ϑ_1) of the project. V_2 occurs during the scoping event (p= 1) of the project 2 weeks after the project started. V_2 is a contract type of variation, k=1. V_2 will impact all the activities that occurred during the first two weeks (ϑ_2) of the project. V_3 occurs during the engineering event (p= 2) of the project 4 weeks after the project started. V_3 is a quality type of variation, k=2. V_3 will impact all the activities that occurred during the first four weeks (ϑ_1) of the project. V_4 occurs during the fabricate event (p= 4) of the project 7 weeks after the project started. V_4 is an uncontrollable type of variation, k=3. V_4 will impact all the activities that occurred during the first seven weeks (ϑ_4) of the project.

For our illustration, we use the Hadamard product (denoted [A] ◦ [B]), which is a binary operation that takes two matrices of the same dimensions, and produces another matrix where each element *ij* is the product of elements *ij* of the original two matrices. It should not be confused with the more common matrix product. For our derivation, we use the symbol "•" to represent multiplication to avoid mistaking the multiplication operator "x" for a variable.

Each variation Δn is expressed as a product of factors associated with the variation type k and is then multiplied by the initial event column matrix [s]. Note that depending on the project and the rework involved, it might be more suitable to use a column matrix other than the initial event column matrix shown in this derivation. This flexibility allows the same method to be customized to fit a variety of projects.

The problem is finding the value or convention for assigning a value to the factors used to operate on the initial event column matrix. A general expression that captures the impact of different types of variation and accounts for the variation impact differences of each event can be expressed using components F_k and M_k. F_k is a multiplier that accounts for the variation type, and M_k is the component that captures the different influence variation has on each event within each type of variation.

$$\Delta_n = \sum_{k=1}^{K} [\{ F_k \bullet M_k \}] \circ [s]_n \qquad \text{Eq. 3.7}$$

The sub notation n in Δ_n and $[s]_n$ is used to denote the variation number. Note that [s] is specific to the variation and is dependent on the time during project progression in which the variation occurs.

For example, the bridge illustration has the notation and values shown in figure 3.7 as follows.

Given $[e]_0 = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 4 \end{bmatrix}$,

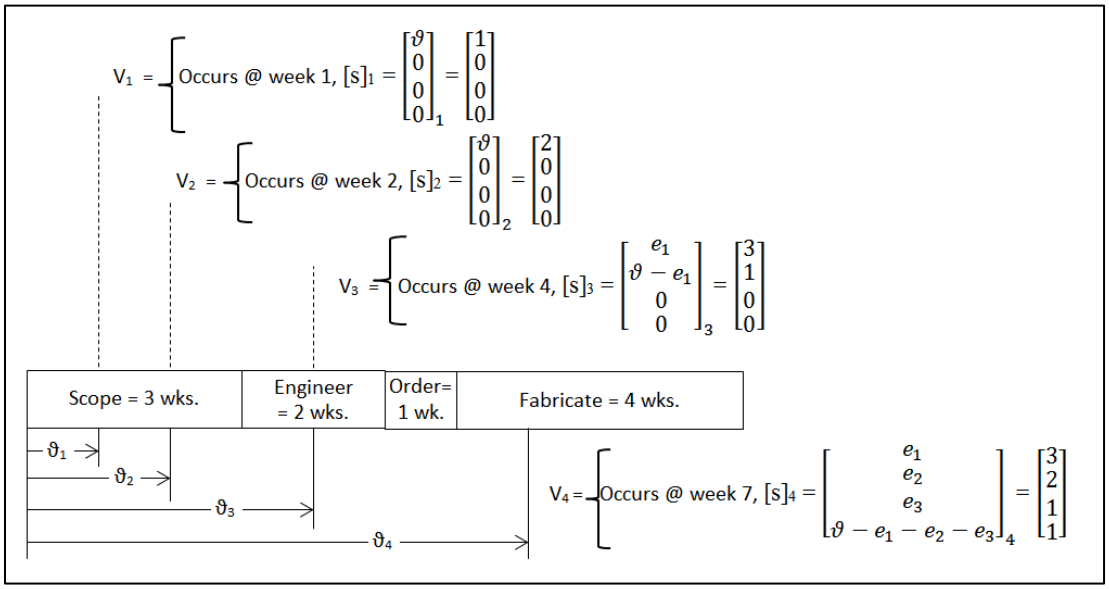


Fig. 3.7 The Notation Used for the Event Matrix $[e]_n$

$$[s]_1 = \begin{bmatrix} \vartheta \\ 0 \\ 0 \\ 0 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, [s]_2 = \begin{bmatrix} \vartheta \\ 0 \\ 0 \\ 0 \end{bmatrix}_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, [s]_3 = \begin{bmatrix} e_1 \\ \vartheta - e_1 \\ 0 \\ 0 \end{bmatrix}_3 = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix},$$

$$[s]_4 = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vartheta - e_1 - e_2 - e_3 \end{bmatrix}_4 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

Let $\mu_k = 1$ if k belongs to the variation, i.e. k is involved in the variation, and $\mu_k = 0$ if k does not belong to the variation.

Then,

$$F_k = \frac{\mu_k}{\sum_{i=1}^{K} \mu_i T_i} T_k \qquad \text{Eq. 3.8}$$

Let $\rho_p = 1$ if event p belongs to the variation, i.e. p is involved in the variation., and $\rho_p = 0$ if p does not belongs to the variation.

$$M_k = \frac{1}{\sum_{p=1}^{P} \rho_p^k R_p^k} [\rho]_k \circ [R]_k \qquad \text{Eq. 3.9}$$

3.6 Algebraic Solution of the Rework Matrix Used on a Sequence of Events

For the bridge illustration, k = 1,2,3. k is the type of change in a design variation

k= 1 means "Contract" change.

k = 2 means "Quality Control" change.

k = 3 means "Uncontrollable" change.

K = 3 is the total types of changes for our bridge illustration.

Also, for the bridge illustration, p = 1,2,3,4. p is the event identification.

p = 1 means "Scoping" event of the project.

p = 2 means "Engineering" event of the project.

p = 3 means "Ordering" event of the project.

p = 4 means "Fabricating" event of the project.

P = 4 is the total number of events in a project.

Expanding Eq. 3.7 gives:

$$\Delta_n = [\{ (F_1 \bullet M_1) + (F_2 \bullet M_2) + (F_3 \bullet M_3) \}] \circ [s]_n \qquad \text{Eq. 3.10}$$

where

$$F_1 = \frac{\mu_1}{(\mu_1 \bullet T_1 + \mu_2 \bullet T_2 + \mu_3 \bullet T_3)} \bullet T_1 \qquad \text{Eq. 3.11}$$

$$F_2 = \frac{\mu_2}{(\mu_1 \bullet T_1 + \mu_2 \bullet T_2 + \mu_3 \bullet T_3)} \bullet T_2 \qquad \text{Eq. 3.12}$$

$$F_3 = \frac{\mu_3}{(\mu_1 \bullet T_1 + \mu_2 \bullet T_2 + \mu_3 \bullet T_3)} \bullet T_3 \qquad \text{Eq. 3.13}$$

and

$$M_1 = \frac{1}{(\rho_1^1 \bullet R_1^1 + \rho_2^1 \bullet R_2^1 + \rho_3^1 \bullet R_3^1 + \rho_4^1 \bullet R_4^1)} \bullet [\rho]_1 \circ [R]_1 \qquad \text{Eq. 3.14}$$

$$M_2 = \frac{1}{(\rho_1^2 \bullet R_1^2 + \rho_2^2 \bullet R_2^2 + \rho_3^2 \bullet R_3^2 + \rho_4^2 \bullet R_4^2)} \bullet [\rho]_2 \circ [R]_2 \qquad \text{Eq. 3.15}$$

$$M_3 = \frac{1}{(\rho_1^3 \bullet R_1^3 + \rho_2^3 \bullet R_2^3 + \rho_3^3 \bullet R_3^3 + \rho_4^3 \bullet R_4^3)} \bullet [\rho]_3 \circ [R]_3 \qquad \text{Eq. 3.16}$$

$[\rho]_1 = , \begin{bmatrix} \rho_1^1 \\ \rho_2^1 \\ \rho_3^1 \\ \rho_4^1 \end{bmatrix}_1$ or, to simplify, because the subscript outside the matrix brackets denotes that the binomial column matrix, $\begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{bmatrix}_1$ belongs to the change type k = 1. Likewise,

$[\rho]_2 = , \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{bmatrix}_2$ and $[\rho]_3 = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{bmatrix}_3$.

$[R]_1 = \begin{bmatrix} R_1^1 \\ R_2^1 \\ R_3^1 \\ R_4^1 \end{bmatrix}_1$ or, to simplify, because the subscript outside the matrix brackets denotes that the binomial column matrix $\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}_1$ belongs to the change type k = 1. Likewise,

$[R]_2 = , \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}_2$ and $[R]_3 = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}_3$.

Note that this general expression can be used for many types of variations with as many numbers of variations required capturing the full variation impact. Also, each variation can have components of each class that the particular variation impacts. The total variation for the four variations used in our illustration is:

$$\text{Total } \Delta = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 \qquad \text{Eq. 3.17}$$

For the bridge illustration, variation 1 occurred 1 week after the project started; the only impact was e1. It is a contract variation that specifies that the bridge be wider than agreed upon in the original contract.

$$\Delta_1 = [\{(F_1 \bullet M_1) + (F_2 \bullet M_2) + (F_3 \bullet M_3)\}] \circ [s]_1 \qquad \text{Eq. 3.18}$$

The variation is a contract variation, which reduces equation 5 to:

$$\Delta_1 = [\{(F_1 \bullet M_1)\}] \circ [s]_1 \qquad \text{Eq. 3.19}$$

Also, recall

$$F_1 = \frac{\mu_1}{(\mu_1 \bullet T_1 + \mu_2 \bullet T_2 + \mu_3 \bullet T_3)} \bullet T_1 \qquad \text{Eq. 3.20}$$

Because there is only a contract variation, μ1 and μ2 = 0, and equation 6 reduces to

$$F_1 = \frac{\mu_1}{(\mu_1 \bullet T_1)} \bullet T_1 \qquad \text{Eq. 3.21}$$

giving

$$F_1 = 1 \qquad \text{Eq. 3.22}$$

Recall equation 9,

$$M_1 = \frac{1}{(\rho_1^1 \bullet R_1^1 + \rho_2^1 \bullet R_2^1 + \rho_3^1 \bullet R_3^1 + \rho_4^1 \bullet R_4^1)} \bullet [\rho]_1 \circ [R]_1 \qquad \text{Eq. 3.23}$$

Only event one is impacted resulting in ρ_2^1, ρ_3^1, and ρ_4^1 = 0. Substituting into equation 9 gives

$$M_1 = \frac{1}{(\rho_1^1 \bullet R_1^1 + 0 \bullet R_2^1 + 0 \bullet R_3^1 + 0 \bullet R_4^1)} \bullet \begin{bmatrix} \rho_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}_1 \circ [R]_1 \qquad \text{Eq. 3.24}$$

and

$$M_1 = \begin{bmatrix} 1/R_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}_1 \circ \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}_1 \qquad \text{Eq. 3.25}$$

This results in

$$M_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}_1 \qquad \text{Eq. 3.26}$$

giving $$\Delta_1 = \{(1 \bullet \begin{bmatrix}1\\0\\0\\0\end{bmatrix}_1)\} \circ \begin{bmatrix}\vartheta\\0\\0\\0\end{bmatrix}_1 \qquad \text{Eq. 3.27}$$

which yields $$\Delta_1 = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}_1 \qquad \text{Eq. 3.28}$$

where $\vartheta = 1$ because the time that has elapsed since project initiation is only 1 week. Note that in equation 3.19, the sub notation outside the matrix brackets specifies the variation number, not the variation type. This is because the event variation impact column matrix [s] is specific to the time in which the variation occurred during project progression.

For the bridge illustration, variation 2 occurred 2 weeks after the project started. Thus, only e1 was impacted. It is a contract variation that specifies that the bridge capacity be more than agreed upon in the original contract. Recall equation 5,

$$\Delta_2 = [\{(F_1 \bullet M_1) + (F_2 \bullet M_2) + (F_3 \bullet M_3)\}] \circ [s]_2 \qquad \text{Eq. 3.29}$$

reduces to $$\Delta_2 = \{(F_1 \bullet M_1)\} \circ [e]_2 \qquad \text{Eq. 3.30}$$

and $$F_1 = \frac{\mu_1}{(\mu_1 \bullet T_1 + \mu_2 \bullet T_2 + \mu_3 \bullet T_3)} \bullet T_1 \qquad \text{Eq. 3.31}$$

Because there is only a contract variation, μ1 and μ2 = 0, and equation reduces to

$$F_1 = \frac{\mu_1}{(\mu_1 \bullet T_1)} \bullet T_1 \qquad \text{Eq. 3.32}$$

giving $$F_1 = 1 \qquad \text{Eq. 3.33}$$

and the equation

$$M_1 = \frac{1}{(\rho_1^1 \bullet R_1^1 + \rho_2^1 \bullet R_2^1 + \rho_3^1 \bullet R_3^1 + \rho_4^1 \bullet R_4^1)} \bullet [\rho]_1 \circ [R]_1 \qquad \text{Eq. 3.34}$$

Only event one is impacted, resulting in ρ_2^1, ρ_3^1, and $\rho_4^1 = 0$. Substituting into equation 3.34 gives

$$M_1 = \frac{1}{(\rho_1^1 \bullet R_1^1 + 0 \bullet R_2^1 + 0 \bullet R_3^1 + 0 \bullet R_4^1)} \bullet \begin{bmatrix} \rho_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}_1 \circ [R]_1 \qquad \text{Eq. 3.35}$$

and

$$M_1 = \begin{bmatrix} 1/R_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}_1 \circ \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}_1 \qquad \text{Eq. 3.36}$$

This results in

$$M_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}_1 \qquad \text{Eq. 3.37}$$

giving

$$\Delta_2 = \{(1 \bullet \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}_1)\} \circ \begin{bmatrix} \vartheta \\ 0 \\ 0 \\ 0 \end{bmatrix}_2 \qquad \text{Eq. 3.38}$$

which yields

$$\Delta_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}_2 \qquad \text{Eq. 3.39}$$

where $e_1 = 2$ because the time that has elapsed since project initiation is only 2 weeks. Note in equation 27 that the sub notation outside the matrix brackets specifies the variation number, not the variation type. This is because the event variation impact column matrix [s] is specific to the time in which the variation occurred during project progression.

For the bridge illustration, variation 3 occurred 4 weeks after the project started impacting e1and e2. It is a quality variation. Recall equation 5,

$$\Delta_3 = [\{(F_1 \bullet M_1) + (F_2 \bullet M_2) + (F_3 \bullet M_3)\}] \circ [s]_3 \qquad \text{Eq. 3.40}$$

The variation is a quality variation, which reduces equation 5 to

$$\Delta_3 = [\{(F_2 \bullet M_2)\}] \circ [s]_3 \qquad \text{Eq. 3.41}$$

Also recall,

$$F_2 = \frac{\mu_2}{(\mu_1 \bullet T_1 + \mu_2 \bullet T_2 + \mu_3 \bullet T_3)} \bullet T_2 \qquad \text{Eq. 3.42}$$

Because there is only a quality variation, μ1 and μ3 = 0, and equation 6 reduces to

$$F_2 = \frac{\mu_2}{(\mu_2 \bullet T_2)} \bullet T_2 \qquad \text{Eq. 3.43}$$

giving

$$F_2 = 1 \qquad \text{Eq. 3.44}$$

Recall equation 9,

$$M_2 = \frac{1}{(\rho_1^2 \bullet R_1^2 + \rho_2^2 \bullet R_2^2 + \rho_3^2 \bullet R_3^2 + \rho_4^2 \bullet R_4^2)} \bullet [\rho]_2 \circ [R]_2 \qquad \text{Eq. 3.45}$$

Only event one and two are impacted, resulting in ρ_3^2 and ρ_4^2 = 0. Substituting into equation 9 gives

$$M_2 = \frac{1}{(\rho_1^2 \bullet R_1^2 + \rho_2^2 \bullet R_2^2 + 0 \bullet R_3^2 + 0 \bullet R_4^2)} \bullet \begin{bmatrix} \rho_1 \\ \rho_2 \\ 0 \\ 0 \end{bmatrix}_2 \circ [R]_2 \qquad \text{Eq. 3.46}$$

and

$$M_2 = \begin{bmatrix} \rho_1/(\rho_1 \bullet R_1 + \rho_2 \bullet R_2) \\ \rho_2/(\rho_1 \bullet R_1 + \rho_2 \bullet R_2) \\ 0 \\ 0 \end{bmatrix}_2 \circ \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}_2 \qquad \text{Eq. 3.47}$$

This results in

$$M_2 = \begin{bmatrix} \rho_1 \bullet R_1/(\rho_1 \bullet R_1 + \rho_2 \bullet R_2) \\ \rho_2 \bullet R_2/(\rho_1 \bullet R_1 + \rho_2 \bullet R_2) \\ 0 \\ 0 \end{bmatrix}_2 \qquad \text{Eq. 3.48}$$

giving

$$\Delta_3 = \{(1 \bullet \begin{bmatrix} \rho_1 \bullet R_1/(\rho_1 \bullet R_1 + \rho_2 \bullet R_2) \\ \rho_2 \bullet R_2/(\rho_1 \bullet R_1 + \rho_2 \bullet R_2) \\ 0 \\ 0 \end{bmatrix}_2)\} \circ \begin{bmatrix} e_1 \\ \vartheta - e_2 \\ 0 \\ 0 \end{bmatrix}_3 \qquad \text{Eq. 3.49}$$

The binomial ρ's = 1, to yield

$$\Delta_3 = \begin{bmatrix} e_1 \bullet R_1^2/(R_1^2 + R_2^2) \\ (\vartheta - e_1) \bullet R_2^2/(R_1^2 + R_2^2) \\ 0 \\ 0 \end{bmatrix}_3 \qquad \text{Eq. 3.50}$$

where $e_1 = 3$ and $\vartheta - e_2 = 1$ because the time that has elapsed since project initiation is 4 weeks. Note that the sub notation outside the matrix brackets specifies the variation number, not the variation type. This is because the event variation impact column matrix [s] is specific to the time in which the variation occurred during project progression.

For the bridge illustration, variation 4 occurred 7 weeks after the project started, and the uncontrollable variation increased the bridge capacity. The variation impacts e1, e2, e3, and e4. Recall equation 5,

$$\Delta_4 = [\{(F_1 \bullet M_1) + (F_2 \bullet M_2) + (F_3 \bullet M_3)\}] \circ [s]_4 \qquad \text{Eq. 3.51}$$

The variation is a quality variation, which reduces equation 5 to

$$\Delta_4 = [\{(F_3 \bullet M_3)\}] \circ [s]_4 \qquad \text{Eq. 3.52}$$

Also, recall

$$F_3 = \frac{\mu_3}{(\mu_1 \bullet T_1 + \mu_2 \bullet T_2 + \mu_3 \bullet T_3)} \bullet T_3 \qquad \text{Eq. 3.53}$$

Because there is only a uncontrollable variation, μ1 and μ2 = 0, and equation 6 reduces to

$$F_3 = \frac{\mu_3}{(\mu_3 \bullet T_3)} \bullet T_3 \qquad \text{Eq. 3.54}$$

giving

$$F_3 = 1 \qquad \text{Eq. 3.55}$$

Recall equation 9,

$$M_3 = \frac{1}{(\rho_1^3 \bullet R_1^3 + \rho_2^3 \bullet R_2^3 + \rho_3^3 \bullet R_3^3 + \rho_4^3 \bullet R_4^3)} \bullet [\rho]_3 \circ [R]_3 \qquad \text{Eq. 3.56}$$

All events are impacted, and the binomial ρ's =1 . Substituting into equation 9 gives

$$M_3 = \frac{1}{(1 \bullet R_1^3 + 1 \bullet R_2^3 + 1 \bullet R_3^3 + 1 \bullet R_4^3)} \bullet \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}_3 \circ [R]_3 \qquad \text{Eq. 3.57}$$

and

$$M_3 = \begin{bmatrix} 1/(R_1 + R_2 + R_3 + R_4) \\ 1/(R_1 + R_2 + R_3 + R_4) \\ 1/(R_1 + R_2 + R_3 + R_4) \\ 1/(R_1 + R_2 + R_3 + R_4) \end{bmatrix}_3 \circ \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}_3 \qquad \text{Eq. 3.58}$$

$$= \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vartheta - e_1 - e_2 - e_3 \end{bmatrix}_4 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

This results in $$M_3 = \begin{bmatrix} R_1/(R_1+R_2+R_3+R_4) \\ R_2/(R_1+R_2+R_3+R_4) \\ R_3/(R_1+R_2+R_3+R_4) \\ R_4/(R_1+R_2+R_3+R_4) \end{bmatrix}_3 \qquad \text{Eq. 3.59}$$

giving $$\Delta_4 = \{(1 \bullet \begin{bmatrix} R_1/(R_1+R_2+R_3+R_4) \\ R_2/(R_1+R_2+R_3+R_4) \\ R_3/(R_1+R_2+R_3+R_4) \\ R_4/(R_1+R_2+R_3+R_4) \end{bmatrix}_3)\} \circ \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vartheta - e_1 - e_2 - e_3 \end{bmatrix}_4 \qquad \text{Eq. 3.60}$$

The sub notation for the matrix M specifies the type of variation and is brought inside the matrix brackets as a superscript. The binomial ρ's = 1, and by definition $R_1+R_2+R_3+R_4 = 1$ substituting yields

$$\Delta_4 = \begin{bmatrix} e_1 \bullet R_1^3 \\ e_2 \bullet R_2^3 \\ e_3 \bullet R_3^3 \\ (\vartheta - e_1 - e_2 - e_3) \bullet R_4^3 \end{bmatrix}_4 \qquad \text{Eq. 3.61}$$

where $e_1 = 3$ and $e_2 = 2$, e3 = 1 and $(\vartheta - e_1 - e_2 - e_3) = 1$ because the time that has elapsed since project initiation is 7 weeks. Note that the sub notation outside the matrix brackets specifies the variation number, not the variation type. This is because the event matrix [s] is specific to the time in which the variation occurred during project progression. Equation 3.6,

$$\text{Total } \Delta = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 \qquad \text{Eq. 3.62}$$

gives

$$\text{Total } \Delta = \begin{bmatrix}\vartheta\\0\\0\\0\end{bmatrix}_1 + \begin{bmatrix}\vartheta\\0\\0\\0\end{bmatrix}_2 + \begin{bmatrix} e_1 \bullet R_1^2/(R_1^2+R_2^2)\\ (\vartheta - e_1)\bullet R_2^2/(R_1^2+R_2^2)\\ 0\\ 0\end{bmatrix}_3 + \begin{bmatrix} e_1 \bullet R_1^3\\ e_2 \bullet R_2^3\\ e_3 \bullet R_3^3\\ (\vartheta - e_1 - e_2 - e_3)\bullet R_4^3\end{bmatrix}_4 \qquad \text{Eq. 3.63}$$

For the bridge illustration, where k is the variation type (k=1 for Contract, k=2 for Quality, k=3 for Uncontrollable), and p is the event during which variation occurs (p=1 for scope, p=2 for engineering, p=3 for ordering, p=4 for fabricating). Table 3.1 below summarizes the algebraic results derived above for a variation occurring at any time during bridge project progression.

Table 3.1. Δ_p^k Time Increase Due to Variation k During Event p

Δ_p^k		Event during which variation occurs			
		p=1	p=2	p=3	p=4
Variation Type	k=1	$\Delta_1^1 = \begin{bmatrix}\vartheta\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix} e_1 \bullet R_1^1/(R_1^1+R_2^1)\\ (\vartheta - e_1)\bullet R_2^1/(R_1^1+R_2^1)\\ 0\\ 0\end{bmatrix}$	$\begin{bmatrix} e_1 \bullet R_1^1/(R_1^1+R_2^1+R_3^1)\\ e_2 \bullet R_2^1/(R_1^1+R_2^1+R_3^1)\\ (\vartheta - e_1 - e_2)\bullet R_3^1/(R_1^1+R_2^1+R_3^1)\\ 0\end{bmatrix}$	$\begin{bmatrix} e_1 \bullet R_1^1\\ e_2 \bullet R_2^1\\ e_3 \bullet R_3^1\\ (\vartheta - e_1 - e_2 - e_3)\bullet R_4^1\end{bmatrix}$
	k=2	$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix} e_1 \bullet R_1^2/(R_1^2+R_2^2)\\ (\vartheta - e_1)\bullet R_2^2/(R_1^2+R_2^2)\\ 0\\ 0\end{bmatrix}$	$\begin{bmatrix} e_1 \bullet R_1^2/(R_1^2+R_2^2+R_3^2)\\ e_2 \bullet R_2^2/(R_1^2+R_2^2+R_3^2)\\ (\vartheta - e_1 - e_2)\bullet R_3^2/(R_1^2+R_2^2+R_3^2)\\ 0\end{bmatrix}$	$\begin{bmatrix} e_1 \bullet R_1^2\\ e_2 \bullet R_2^2\\ e_3 \bullet R_3^2\\ (\vartheta - e_1 - e_2 - e_3)\bullet R_4^2\end{bmatrix}$
	k=2	$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix} e_1 \bullet R_1^3/(R_1^3+R_2^3)\\ (\vartheta - e_1)\bullet R_2^3/(R_1^3+R_2^3)\\ 0\\ 0\end{bmatrix}$	$\begin{bmatrix} e_1 \bullet R_1^3/(R_1^3+R_2^3+R_3^3)\\ e_2 \bullet R_2^3/(R_1^3+R_2^3+R_3^3)\\ (\vartheta - e_1 - e_2)\bullet R_3^3/(R_1^3+R_2^3+R_3^3)\\ 0\end{bmatrix}$	$\begin{bmatrix} e_1 \bullet R_1^3\\ e_2 \bullet R_2^3\\ e_3 \bullet R_3^3\\ (\vartheta - e_1 - e_2 - e_3)\bullet R_4^3\end{bmatrix}$

3.7 Results

The key attribute of equations 3.1b, 3.2 and 3.7 is that they can be used for as many variations and as many variation types within each variation required to model the actual project disruptions. The flexibility of equations 3.1b, 3.2 and 3.7 is another advantage allowing a variety of techniques to be used for assigning values to F_k and the elements of the M_k matrix. The method suggested and discussed in the next chapter is one of many approaches to consider. For example, the matrices could be populated with elements that are time dependent equations or a simpler convention such as using integers for multiplying the initial project timeline. Large projects have hundreds or even thousands of events resulting in an overwhelming amount of data. A computer program is necessary for handling such a large amount of data efficiently. A powerful feature of equations 3.1b, 3.2 and 3.7 is the capacity for handling large quantities of data by utilizing matrices and binomial variables μ and ρ which is a convenient format required for computer programming. From the illustration in section 3.1, the equation that accounts for variation impact has a sub notation n on the outside of each matrix to designate the variation impact calculated.

$$[e]_n + [CM]_n = [E]_n \qquad \text{Eq. 3.1b}$$

The sub notation 0 is used for the original project plan, and for the bridge illustration it has the following notation and values.

$$\begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \\ 4 \end{bmatrix}_0 + \begin{bmatrix} 0 \\ 3 \\ 5 \\ 6 \end{bmatrix}_0 = \begin{bmatrix} 3 \\ 5 \\ 6 \\ 10 \end{bmatrix}_0 \qquad \text{Eq. 3.1c}$$

The event duration matrix for variation n is expressed as

$$[e]_n = [e]_0 + Total\ \Delta \qquad \text{Eq. 3.64}$$

where $[e]_0$ is the initial event matrix and, because this is single series of events,

$$\text{Total } \Delta = \begin{bmatrix} \Delta t_1 \\ \Delta t_2 \\ \Delta t_3 \\ \Delta t_i \end{bmatrix}_1 + \begin{bmatrix} \Delta t_1 \\ \Delta t_2 \\ \Delta t_3 \\ \Delta t_i \end{bmatrix}_2 + \ldots \begin{bmatrix} \Delta t_1 \\ \Delta t_2 \\ \Delta t_3 \\ \Delta t_i \end{bmatrix}_n \qquad \text{Eq. 3.65}$$

Each variation of n can be used in the equation 3.1b to solve for $[E]_n$. These column matrices can be added in matrix form to examine the variation impact on each event. Note that if there were more than one series of events, and the series were coupled, the variation impacts the duration matrix $[\mathbf{e}]_n$ from equation 3.64, and the other matrices $[CM]_n$ and $[E]_n$ are calculated from equation 3.2 introduced earlier.

Another way to express the variation impact is by examining the difference in project completion time. The change in project completion time is calculated using [E]n from equation 3.2 and finding the difference in the maximum element of $[E]_0$ and the maximum element in $[E]_n$.

$$\Delta E_n = \{(\text{Max } [Ej]_n)_{\ j=1,2,3\ldots P}\} - \{(\text{Max}[E_j]_0)_{\ j=1,2,3\ldots P}\} \qquad \text{Eq. 3.66}$$

CHAPTER 4

SAMPLE CALCULATIONS OF RANKING FOR PARAMETERS

4.1 Analytical Hierarchy Process (AHP)

In the previous chapters, an equation was developed to calculate the amount of time required for reworking an event based on the initial time of that particular event. The product of two factors is used to adjust the initial time for calculating the time required to rework due to variation. The value of the terms F and M are based on the evaluation of a number of alternatives in terms of appropriate criteria. The pertinent data needed to assign values to F and M are difficult to quantify. While there are various ways to assign values to F and M, the one illustrated in this dissertation is the analytical hierarchy process (AHP) because of its efficiency for correctly quantifying the data associated with the interpreting F and M. The Analytical Hierarchy Process (AHP) decomposes problems into a hierarchy of criteria and alternatives. This process has been examined, analyzed, and inspected since Thomas L. Saaty developed the process in the 1970s. The analytical hierarchy decomposition is a three step process. The first step is to state the objective, the second is to define the criteria, and the third is to pick the alternatives. After the decomposition, the next step in the process is to rank the decisions based on several criteria. These rankings are set up in a pairwise matrix, and the right hand eigenvector of the associated pairwise matrices gives the ranking of each alternative based on the established criteria.

The analytical hierarchy process is analyzed, utilized and studied in many countries around the world. For example, China has nearly a hundred universities that offer courses in AHP; over 900 papers have been published on the subject and at least one professional journal focuses on AHP (Saaty, et al.1990). AHP is a tool for decision makers to rank alternatives based

on certain criteria. A key element of the Analytical Hierarchy Process is assigning values to the decision options available in a project or process. These assigned values are used for pairwise comparisons and, ultimately, calculating the eigenvector that ranks decision options relative to a set of criteria. Assigning the correct values to each decision is the most important step in the Analytical Hierarchy Process. Each decision option value is compared to another in light of an established criteria in a process called pairwise comparison. One of the most significant scales used to assign values to each choice developed by Saaty is shown in Table 4.1 (Triantaphyllou, et al.1994). Other scales are suggested in (Triantaphyllou, et al.1994) based on theory. The scale proposed by Saaty established 9 as the scale's upper limit based on Weber's law concerning change of perceptible consequence. In 1846 Weber proposed that a change is perceptible only if it is a constant percentage of the change itself (Saaty 1980). Stated in simpler terms, people can only discern or choose one option from another if the differences between those options or choices are large enough to be discernible by the human senses. Considering an extreme example, people cannot choose between options if the number of options exceeds human perception, such as an infinite number of options. People are not able to consistently discern between an option A given a weight or importance of 5 and option B having a weight of 5.001. To further expound and support the reasoning behind the scale proposed by Saaty, experiments have shown that people cannot mentally process comparing more than seven objects at one time, plus or minus two (Miller, 1956). The data and research performed by Weber and Miller established the foundation for Saaty's scale having an upper limit of 9 and a lower limit of 1. Saaty's scale defines the available values for pairwise comparison as {9,8,7,6,5,4,3,2,1, ½,1/3,1/4,1/5,1/6,1/7,1/8,1/9}.

Before AHP is explained utilizing an illustration, the criticisms of AHP, other multiple decision criteria methods, and the different scales used for multiple decision criteria judgment are discussed. Detailed explanations of these issues are beyond the scope of this dissertation but are mentioned to provide references for further study.

Many colleges teach the Analytical Hierarchy Process. Managers, equipped with an understanding of the foundations of each fundamental element, substantially utilize AHP for operating institutions and businesses (Forman, 2001). There are many that criticize the process even though it is largely accepted as being theoretically sound and abundantly applicable (McCaffrey, 2005). Most of the disapproval arises from an aspect of the process that causes erroneous results due to rank reversal. Several other multiple decision criteria methods besides the AHP are vulnerable to rank reversals, and there are different types of rank reversals (Forman, 2001; Saaty, 2001). The topic of rank reversals is beyond the scope of this dissertation, but it is thoroughly explained in an Operations Research paper (Forman, 2001) and in a chapter entitled Rank Preservation and Reversal. (Saaty, 2001; Dyer and Wendell (1985). Another criticism by Oskar Perron is that the principal right eigenvector method solution of some pairwise matrices is not monotonic (consisting of sets such that each set contains the preceding set or such that each set is contained in the preceding set.(Landau, 1914). This is also true for some reciprocal n x n matrices, where n>3. Several other solution methods are explained in Zermelo, 1928; Hasse, 1961; Ramanujacharyulu, 1964. See Appendix B for pairwise consistency verification.

Several multi-criteria decision-making methods have been proposed as an improvement to Saaty's AHP. In 1983, Belton and Gear illustrated that the AHP can reverse the ranking of the choices when an identical choice to one of the initial choices is incorporated (Belton and Gear, 1983). To avoid reverse ranking, they suggest that the elements in each column of the AHP

decision matrix be divided by the maximum element in that column. This suggested approach was called the revised-AHP, and later, after Saaty's acceptance, it was called the Ideal Mode AHP (Saaty's 1994). Other interested parties also introduced AHP variants. In 1993, Forman proposed an AHP synthesis mode to fix a problem in which adding or removing an extraneous choice will not change the ranks of the initial choices (Forman, 1993). The updated AHP version accommodates rank preservation using the ideal mode and allows rank to change using the distributive mode. The AHP problem type dictates the mode used to calculate the solution. Presently, the AHP both in the original or ideal mode is the most universally approved practice and is treated by many as the most dependable multi-criteria decision-making method (Triantaphyllou et al. 1994).

The primary obstacle of the pairwise comparisons used in Saaty's Analytical Hierarchy Process is evaluating the available alternatives. Every multi-criteria decision-making technique that uses the pairwise comparisons approach ultimately assigns ratios of integers to the decision maker's qualitative descriptions of each alternative. This method can be used to define correlation associations (Triantaphyllou 1993). Saaty proposed the scale shown in Table 4.1. Other scales have also been proposed such as the 78 different scales evaluated by Triantaphyllou (Triantaphyllou et al. 1994). All the different scales are rooted from some scholarly rationale that establish the scheme used to assign quantities to the annotated alternatives. These references for different scales used in multi-criteria decision-making methods can be used for further study.

4.2 AHP Used to Rank Variation Classifications

For our example, the objective is to define the rework factor for each event. The criterion is based on the variation caused by contract changes, quality discrepancies or uncontrollable events. Variation causes contract changes because of technology changes, customer requests, or

miscommunication in the original contract. Quality discrepancies could be bad material from the supplier, trades workmanship, or miscommunication. Another variation criterion is uncontrollable events such as changes in government legislation, natural disasters, or radical global market changes. The alternatives are the activities being performed.

Problems are decomposed into a hierarchy of criteria and alternatives using the following steps:

State the objective

• Define the weighting of each variation classification as it applies to each type of event

• The impact of variation on project events such as scoping, engineering, ordering material, and fabricating.

Pick the alternatives

• Contract Variations, Quality variations, Uncontrollable variations

Given a hierarchy shown in figure 4.1 below, and a scale for pairwise comparison shown in table 4.1, a valuation of events relative to the variation type can be made as shown in table 4.2. Then the pairwise comparison can be made as shown in table 4.3.

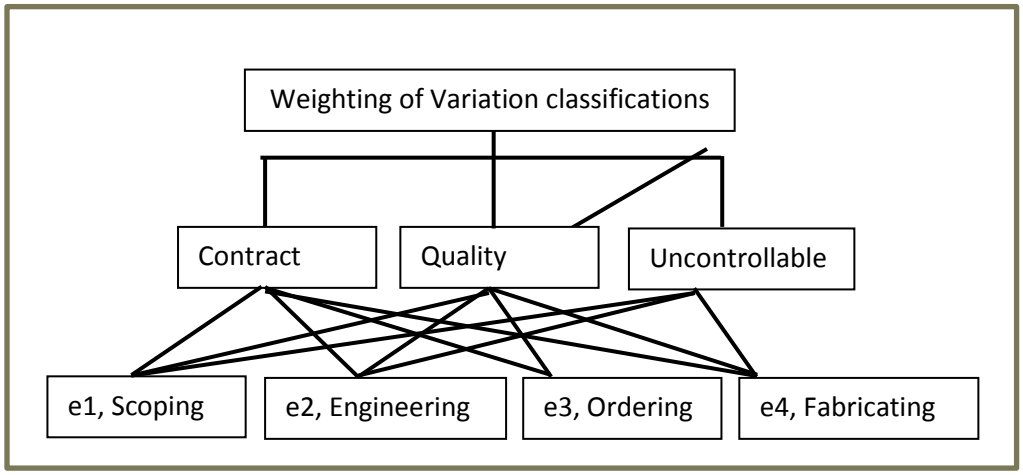


Figure 4.1. Hierarchy

Table 4.1. Scale for Pairwise Comparison

The Scale used for Pairwise Comparison		
Magnitude of Significance	**Description**	**Interpretation**
1	Equivalent Weight	Variation classifications or within two events have the same project impact.
3	Modest Weight	Experience evaluates one variation classification or variation within an event moderately higher than another.
5	Heavy Weight	Experience evaluates one variation classification or variation within an event much higher than another.
7	Very Heavy Weight	One variation classification or variation within an event is evaluated as being very much higher than another.
9	Maximum Weight	The history of experience evaluates one variation classification or variation within an event as having the highest possible project impact.
Weights of 1.1, 1.2, and 1.3 are used for variation classification or variation within an event that have only a slightly different weight. Even magnitude such as 2,4,6, and 8 are for assigning intermediate weights.		

Table 4.2. Evaluating Events Relative to Variation Type

Event	**Variation Type**		
	Contract	**Quality**	**Uncontrollable**
e_1, scoping the foundation	Any contract change will involve some amount of scoping to evaluate the extent that project activities will be affected.	Quality problems require a thorough investigation or scoping to discover the extent and magnitude of the effects to other project activities.	Uncontrollable changes are often defined minimizing the amount of scoping required to evaluate the modifications needed from each activity to complete the project.
e_2, engineering the foundation	Most contract changes require engineering to re-analyze the contract modified project to ensure the revised product meets customer criteria.	The engineering department must be notified of any quality problems so that a thorough product analysis can be performed to uncover any potential problems that might occur.	The uncontrollable changes that involve design changes will involve design changes will involve engineering, while others, such as pricing and scheduling, will not require engineering involvement.
e_3, ordering the foundation material	Unless a contract change requires a different component, material, or modifies a suppliers lead time, then purchasing would not be involved in the change.	Many quality problems require a supplier to replace parts or service a warranty. Warranty issues and supplier accountability are usually handled by the purchasing department	Many quality problems occur during product fabrication. Problems such as this need to be reworked or recalled to perform corrections.
e_4, installing the foundation	Contract changes usually occur during the beginning of the project allowing enough lead time for adjustment to avoid any adverse implication to the fabrication process	Fabrication will almost always be affected adversely due to quality problems.	Uncontrollable changes can affect the fabrication process, although changes such as pricing can have minimal fabrication process impact.

Table 4.3. Pairwise Comparison Variation Type Relative to Each Other

Ranking		Explanation
Contract = 6	Quality = 2	Contract changes occur frequently while there are procedures in place for quality control that limits their occurrence and impact.
Contract = 2	Uncontrollable = 1	Contract changes occur frequently while uncontrollable events are limited.
Quality = 2	Uncontrollable = 1	There are procedures in place for quality control that limit their occurrence, while uncontrollable events are limited.

That gives a pairwise matrix of

$$\begin{array}{l} \\ \text{Contract} \\ \text{Quality} \\ \text{Uncontrollable} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Contract} & \text{Quality} & \text{Uncontrollable} \end{array} \\ \begin{pmatrix} 1 & 3 & 2 \\ 1/3 & 1 & 2 \\ 1/2 & 1/2 & 1 \end{pmatrix} \end{array}$$

Solving results for their relative rework weights, (see appendix A for Eigen value calculations)

$$[\,T_{\text{Criteria}}\,] = \begin{pmatrix} 0.5472 \\ 0.2631 \\ 0.1897 \end{pmatrix}$$

4.3 AHP Used to Rank Events

In terms of rework due to variation caused by a request that changes the contract, pairwise comparisons determine the preference of each alternative over another. Table 4.4 shows the rework due to variation caused by a request to change the contract.

Table 4.4. Evaluating Events Relative to Contract Variation

Event	**Contract**
e1, scoping the foundation	Any contract change will involve some amount of scoping to evaluate the extent that project activities will be affected.
e2, engineering the foundation	Most contract changes require engineering to re-analyze the contract modified project to ensure the revised product meets customer criteria.
e3, ordering the material for the foundation	Unless a contract change requires a different component, material, or modifies a suppliers lead time, then purchasing would not be involved in the change.
e4, installing the foundation	Contract changes usually occur during the beginning of the project allowing enough lead time for adjustment to avoid any adverse implication to the fabrication process

In table 4.5 each activity is ranked for setting up the pairwise comparison matrix.

Table 4.5. Ranking Events Relative to Contract Variation

Ranking		Explanation
$e_1 = 2$	$e_2 = 1$	A contract change has more impact on scoping than engineering
$e_1 = 6$	$e_3 = 1$	A contract change has more impact on scoping than ordering
$e_1 = 3$	$e_4 = 2$	A contract change has more impact on scoping than fabricating
$e_2 = 4$	$e_3 = 1$	A contract change has more impact on engineering than ordering
$e_2 = 1$	$e_4 = 4$	A contract change has less impact on engineering than fabricating
$e_3 = 1$	$e_4 = 6$	A contract change has less impact on ordering than fabricating

The ranking in Table 4.5 results in a pairwise matrix of

$$\begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{pmatrix} 1 & 2 & 6 & 3/2 \\ 1/2 & 1 & 4 & ¼ \\ 1/6 & 1/4 & 1 & 1/6 \\ 2/3 & 4 & 6 & 1 \end{pmatrix} \end{array}$$

The values in the matrix's right eigenvector are measurements of their relative rework weights.

Solving results in:

$$[\,R\,]_{\text{Contract}} = \begin{pmatrix} .392 \\ .162 \\ .054 \\ .392 \end{pmatrix}$$

Table 4.6 evaluates the rework due to variation caused by quality.

Table 4.6. Evaluating Events Relative to Quality Variation

Event	**Quality**
e1, scoping the foundation	Quality problems require a thorough investigation or scoping to discover the extent and magnitude of the effects to other project activities.
e2, engineering the foundation	The engineering department must be notified of any quality problems so that a thorough product analysis can be performed to uncover any potential problems that might occur.
e3, ordering the material for the foundation	Many quality problems require a supplier to replace parts or warranty service. Warranty issues and supplier accountability are usually handled by the purchasing department
e4, installing the foundation	Fabrication will almost always be affected adversely due to quality problems.

Ranking each activity for setting up the pairwise comparison matrix gives table 4.7.

Table 4.7. Ranking Events Relative to Quality Variation

Ranking		Explanation
$e_1 = 3$	$e_2 = 1$	A quality change has more impact on scoping than engineering
$e_1 = 6$	$e_3 = 1$	A quality change has more impact on scoping than ordering
$e_1 = 4$	$e_4 = 1$	A quality change has more impact on scoping than fabricating
$e_2 = 2$	$e_3 = 1$	A quality change has more impact on engineering than ordering
$e_2 = 1$	$e_4 = 3$	A quality change has less impact on engineering than fabricating
$e_3 = 1$	$e_4 = 2$	A quality change has less impact on ordering than fabricating

Table 4.7 gives a pairwise matrix of

$$\begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{pmatrix} 1 & 3 & 6 & 4 \\ 1/3 & 1 & 2 & 1/3 \\ 1/6 & 1/2 & 1 & ½ \\ 1/4 & 3 & 2 & 1 \end{pmatrix} \end{array}$$

The priorities are measurements of their relative rework weights; mathematically, they are the values in the matrix's right eigenvector. Solving results in

$$[\,R\,]_{\text{Quality}} = \begin{pmatrix} .560 \\ .134 \\ .084 \\ .222 \end{pmatrix}$$

In terms of rework due to variation caused by uncontrollable, pairwise comparison determines the preference of each alternative over another.

Table 4.8 evaluates the rework due to variation caused by uncontrollable variation.

Table 4.8. Evaluating Events Relative to Uncontrollable Variation

Event	**Uncontrollable**
e1, scoping the foundation	Uncontrollable changes are often defined minimizing the amount of scoping required to evaluate the modifications needed from each activity to complete the project.
e2, engineering the foundation	The uncontrollable changes that involve design changes will involve design changes will involve engineering, while others, such as pricing and scheduling, will not require engineering involvement.
e3, ordering the material for the foundation	Many quality problems occur during product fabrication. Problems such as this need to be reworked or recalled to perform corrections.
e4, installing the foundation	Uncontrollable changes can affect the fabrication process, although changes such as pricing can have minimal fabrication process impact.

Ranking each activity for setting up the pairwise comparison matrix gives table 4.9.

Table 4.9. Ranking Events Relative to Uncontrollable Variation

Ranking		Explanation
$e_1 = 3$	$e_2 = 1$	A uncontrollable change has more impact on scoping than engineering
$e_1 = 4$	$e_3 = 1$	A uncontrollable change has more impact on scoping than ordering
$e_1 = 3$	$e_4 = 1$	A uncontrollable change has more impact on scoping than fabricating
$e_2 = 1$	$e_3 = 2$	A uncontrollable change has more impact on engineering than ordering
$e_2 = 1$	$e_4 = 4$	A uncontrollable change has less impact on engineering than fabricating
$e_3 = 1$	$e_4 = 6$	A uncontrollable change has less impact on ordering than fabricating

Gives a pairwise matrix of

$$\begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{pmatrix} 1 & 3 & 4 & 3 \\ 1/3 & 1 & 1/2 & 1/4 \\ 1/4 & 2 & 1 & 1/6 \\ 1/3 & 4 & 6 & 1 \end{pmatrix} \end{array}$$

The priorities are measurements of their relative rework weights; mathematically, they are the values in the matrix's right eigenvector. Solving results in

$$[\,R\,]_{\text{Uncontrolable}} \begin{pmatrix} .484 \\ .086 \\ .102 \\ .320 \end{pmatrix}$$

Figure 4.2 summarizes the results.

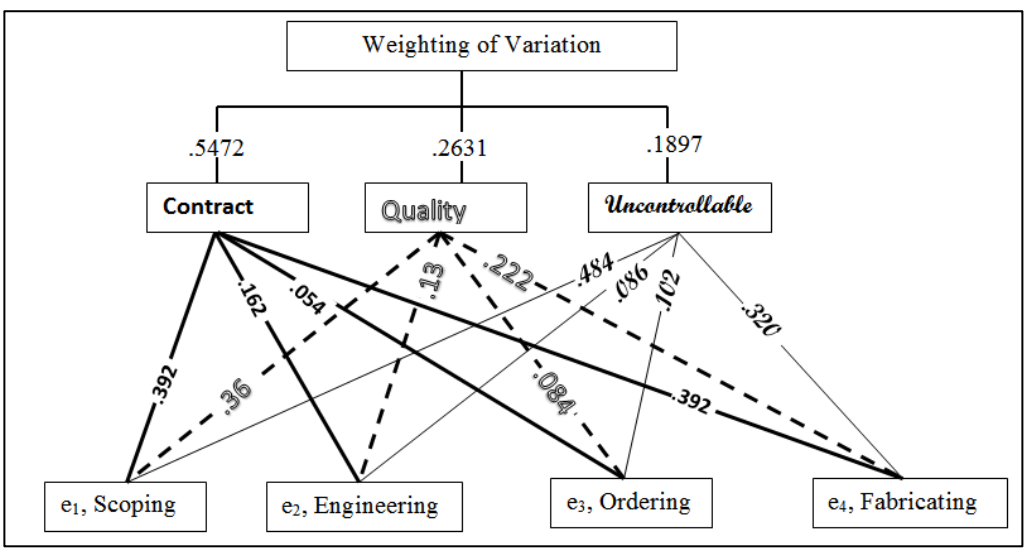


Fig. 4.2. Hierarchy Values

CHAPTER 5

APPLICATIONS

5.1 Utilizing the Derived Equations on a Sequence of Events

The total variation for the four variations used in our bridge illustration is

$$\text{Total } \Delta = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 \qquad \text{Eq. 5.1}$$

Also, from the bridge illustration and our AHP analysis $T_1 = .5472$, T2 = .2631 , T3 = .1897

and $[R]_1 = \begin{pmatrix} .392 \\ .162 \\ .054 \\ .392 \end{pmatrix}_1$ $[R]_2 = \begin{pmatrix} .560 \\ .134 \\ .084 \\ .222 \end{pmatrix}_2$ and $[R]_3 = \begin{pmatrix} .484 \\ .086 \\ .102 \\ .320 \end{pmatrix}_3$.

Also, recall $[e_0] = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 4 \end{pmatrix}$, and

$$\text{Total } \Delta = \begin{pmatrix} e_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}_1 + \begin{pmatrix} e_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}_2 + \begin{pmatrix} e_1 \bullet R_1^2 / (R_1^2 + R_2^2) \\ e_2 \bullet R_2^2 / (R_1^2 + R_2^2) \\ 0 \\ 0 \end{pmatrix}_3 + \begin{pmatrix} e_1 \bullet R_1^3 \\ e_2 \bullet R_2^3 \\ e_3 \bullet R_3^3 \\ e_4 \bullet R_4^3 \end{pmatrix}_4 \qquad \text{Eq. 3.66}$$

Substituting

$$\text{Total } \Delta = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}_1 + \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}_2 + \begin{pmatrix} 3 \bullet .560/(.560 + .134) \\ 1 \bullet .134/(.560 + .134) \\ 0 \\ 0 \end{pmatrix}_3 + \begin{pmatrix} 3 \bullet .484 \\ 2 \bullet .086 \\ 1 \bullet .102 \\ 1 \bullet .320 \end{pmatrix}_4 \qquad \text{Eq. 5.2}$$

gives

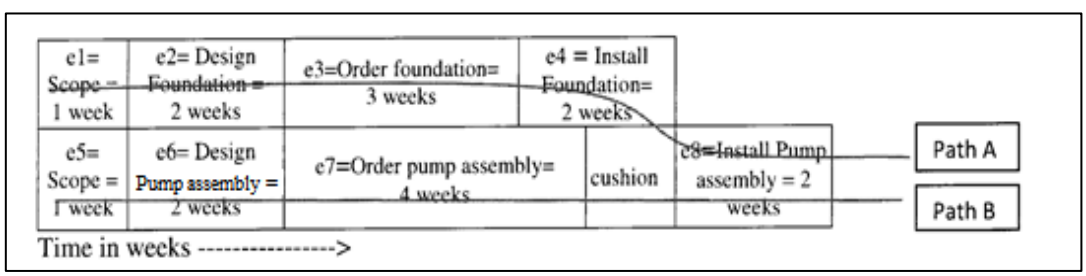
$$\text{Total } \Delta = \begin{pmatrix} 6.873 \\ .365 \\ .102 \\ .320 \end{pmatrix}$$

5.2 Results

There was only one type of variation for each variation used in the example above. This simplifies the equations considerably. Observe in variation three how the equation adjusts for ranking magnitude of the two events impacted by multiplying event one by $\frac{R_1^2}{R_1^2+R_2^2}$ and event two by $\frac{R_2^2}{R_1^2+R_2^2}$. Also, in variation four, all the events are impacted by the variation because of the time the variation occurred. The final total shows that most of the impact was on event one. Intuitively, this result seems correct because every variation occurred after event one had started or was completed.

5.3 Utilizing the Derived Equations on Two Parallel Sequence of Events

Next, consider the two parallel sequential events that are coupled, introduced in chapter 3. The coupling is due to the constraint of requiring the foundation to be finished before the pump assembly can be installed. The two critical paths are copied below from chapter 3 in figure 3.4 for convenience.



In this illustration, the two sequences of events are coupled by requiring the pump foundation (e_4) to be completed before the pump installation ($e_{8)}$. The precedence matrix for this coupling is shown below along with corresponding event equation.

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 3 \\ 6 \\ 0 \\ 1 \\ 3 \\ 8 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 6 \\ 8 \\ 1 \\ 3 \\ 7 \\ 10 \end{pmatrix} \qquad \text{Eq. 5.3}$$

Consider a variation (n=1) at week 4 that has two variation types active (K=3, but $\mu_1 = 1$, $\mu_2 = 0$ and $\mu_3 = 1$). The active variation types are contract (k=1) and uncontrollable (k=3) variation.

Week 7 .Use the same AHP values for both series of events.

Recall Eq. 3.7, Eq. 3.8, and Eq. 3.9 from chapter 3,

$$\Delta_n = \sum_{k=1}^{K} [\{ F_k \bullet M_k \}] \circ [s]_n \qquad \text{Eq. 3.7}$$

$$F_k = \frac{\mu_k}{\sum_{i=1}^{K} \mu_i T_i} T_k \qquad \text{Eq. 3.8}$$

$$M_k = \frac{1}{\sum_{p=1}^{P} \rho_p^k R_p^k} [\rho]_k \circ [R]_k \qquad \text{Eq. 3.9}$$

$$\text{For } \Delta_1 = \sum_{k=1}^{3} (F_k \bullet M_k) \circ [s]_1 \qquad \text{Eq. 5.4}$$

This gives $\Delta_1 = (F1 \bullet M1 + F2 \bullet M2 + F3 \bullet M3) \circ [s]_1$ Eq. 5.5

The F2 • M2 reduce to 0 because μ2 is 0, and μ1, μ3, ρ1 through ρ3 are 1. Equation 3 yields

$$F_1 = \frac{1}{T_1 + T_3} T_1 \qquad \text{Eq. 5.6}$$

and $F_3 = \frac{1}{T_1 + T_3} T_3$ Eq. 5.7

is used for both event sequences.

$$M_1 = \frac{1}{(R_1^1 + R_2^1 + R_3^1 + R_4^1)} \bullet \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}_1 \circ [R]_1 \qquad \text{Eq. 5.8}$$

and

$$M_1 = \begin{pmatrix} 1/(R_1 + R_2 + R_3 + R_4) \\ 1/(R_1 + R_2 + R_3 + R_4) \\ 1/(R_1 + R_2 + R_3 + R_4) \\ 1/(R_1 + R_2 + R_3 + R_4) \end{pmatrix}_1 \circ \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}_1 \qquad \text{Eq. 5.9}$$

By definition (R_1+R_2+R_3+R_4) = 1; therefore,

$$M_1 = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}_1 \qquad \text{Eq. 5.10}$$

For the second sequence, the F2 • M2 also reduces to 0 because μ2 is 0. Note that ρ4 is also 0 because the variation occurred at week 7, and in the second sequence of events the pump installation does not start until after week 7. As noted earlier, the second sequence uses the same AHP results as the first sequence of events. The binomial variables μ1, μ3, and ρ1 through ρ3 are 1; therefore, equation 3 yields

$$M_3 = \frac{1}{(R_1^3 + R_2^3 + R_3^3)} \bullet \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}_3 \circ \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_3 \end{bmatrix}_3 \qquad \text{Eq. 5.11}$$

and

$$M_3 = \begin{pmatrix} 1/(R_1 + R_2 + R_3) \\ 1/(R_1 + R_2 + R_3) \\ 1/(R_1 + R_2 + R_3) \\ 1/(R_1 + R_2 + R_3) \end{pmatrix}_3 \circ \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ 0 \end{bmatrix}_3 = \begin{pmatrix} R_1/(R_1 + R_2 + R_3) \\ R_2/(R_1 + R_2 + R_3) \\ R_3/(R_1 + R_2 + R_3) \\ 0 \end{pmatrix}_3 \qquad \text{Eq. 5.12}$$

The results for sequence one and two are arranged to be used with the precedence matrix shown in equation to give a column matrix of

$$M_1 = \begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_1/(R_1+R_2+R_3) \\ R_2/(R_1+R_2+R_3) \\ R_3/(R_1+R_2+R_3) \\ 0 \end{pmatrix}_1 \qquad \text{Eq. 5.13}$$

and

$$M_3 = \begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_1/(R_1+R_2+R_3) \\ R_2/(R_1+R_2+R_3) \\ R_3/(R_1+R_2+R_3) \\ 0 \end{pmatrix}_3 \qquad \text{Eq.5.14}$$

Also,

$$\Delta_1 = \left(\frac{T_1}{T_1+T_3} \bullet \begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_1/(R_1+R_2+R_3) \\ R_2/(R_1+R_2+R_3) \\ R_3/(R_1+R_2+R_3) \\ 0 \end{pmatrix}_1 + \frac{T_3}{T_1+T_3} \bullet \begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_1/(R_1+R_2+R_3) \\ R_2/(R_1+R_2+R_3) \\ R_3/(R_1+R_2+R_3) \\ 0 \end{pmatrix}_3 \right) \circ \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \\ 0 \end{bmatrix}_1 \qquad \text{Eq. 5.15}$$

Note that the e_4 is 1 because only 1 week of the 2 has occurred.

$\frac{T_1}{T_1+T_3}$ = (.5472)/(.5472+.1897) = .74, and $\frac{T_3}{T_1+T_3}$ = (.1897)/(.5472+.1897) = .26

For Type 1 variations,

$R_1/(R_1+R_2+R_3)$ = .392/(.392+.162+.054) = .64,

$R_2/(R_1+R_2+R_3)$ = .162/(.392+.162+.054) = .27,

$R_3/(R_1+R_2+R_3)$ = .054/(.392+.162+.054) = .09,

For Type 3 variations,

$R_1/(R_1 + R_2 + R_3)$ = .484/(.484+.086+.102) = .72,

$R_2/(R_1 + R_2 + R_3)$ = .086/(.484+.086+.102) = .13,

$R_3/(R_1 + R_2 + R_3)$ = .102/(.484+.086+.102) = .15,

$$\Delta_1 = (.74 \bullet \begin{pmatrix} .392 \\ .162 \\ .054 \\ .392 \\ .640 \\ .270 \\ .090 \\ 0 \end{pmatrix}_1 + .26 \bullet \begin{pmatrix} .484 \\ .086 \\ .102 \\ .320 \\ .720 \\ .130 \\ .150 \\ 0 \end{pmatrix}_3) \circ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \\ 1 \\ 2 \\ 4 \\ 0 \end{pmatrix}_1 = \begin{pmatrix} .416 \\ .284 \\ .199 \\ .373 \\ .660 \\ .467 \\ .422 \\ 0 \end{pmatrix}_1 \qquad \text{Eq. 5.16}$$

Recall,

$$[e]_n = [e]_0 + Total\ \Delta \qquad \text{Eq. 3.64}$$

This gives

$$[e]_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 4 \\ 2 \end{pmatrix}_1 + \begin{pmatrix} .416 \\ .284 \\ .199 \\ .373 \\ .660 \\ .467 \\ .422 \\ 0 \end{pmatrix}_1 = \begin{pmatrix} 1.416 \\ 2.284 \\ 3.199 \\ 2.373 \\ 1.660 \\ 2.467 \\ 4.422 \\ 2 \end{pmatrix}_1 \qquad \text{Eq. 5.18}$$

The final completion matrix is filled from the top cell to the bottom using the equation below to fill the cumulative matrix from chapter 3.

$$[CMi] = \begin{cases} \text{if } i = 1,\ CM_1 = 0 \\ \text{Max} \left\{ \begin{matrix} IF(Pmij = 1, Ej, 0) \\ j=1,2,3\ldots.i-1 \end{matrix} \right\} \text{if } i \neq 1 \end{cases} \qquad \text{Eq. 3.2}$$

The new equation becomes

$$[e]_1 + [CM]_1 = [E]_1 \qquad \text{Eq.5.19a}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1.416 \\ 2.284 \\ 3.199 \\ 2.373 \\ 1.66 \\ 2.467 \\ 4.422 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1.416 \\ 3.7 \\ 6.899 \\ 0 \\ 1.66 \\ 4.127 \\ 9.272 \end{pmatrix} = \begin{pmatrix} 1.416 \\ 3.7 \\ 6.899 \\ 9.272 \\ 1.66 \\ 4.127 \\ 8.549 \\ 11.272 \end{pmatrix} \quad \text{Eq. 5.19b}$$

5.4 Results

The maximum element in $[E_j]$ is 11.272; thus, the final project completion time after variation Δ_1 is 11.272 weeks. This is a 1.272 increase from the original project completion time of 10 weeks. Examining each series separately, equation 5.18 gives a variation impact of 1.27 and 1.55 for the first and second series respectively. Intuitively, it would seem the final project completion time would be the larger of the two series, which is the second series of 9 weeks plus a variation impact of 1.55 weeks giving a total of 10.55 weeks, but the two series of events are coupled by e_4 having to be completed before e_8 shown in the precedence matrix. Initially, the first series of events was 8 weeks, and the first three events in the second series were 7 weeks. This resulted in a waiting period of 1 week. After the variation, the time to complete the first four events is 9.272 (see E_4 in the completion matrix above). The time to complete the first three events in series two is 8.549 weeks. This results in a waiting period of .723 weeks. Equation 3.2 accounts for the coupling by populating element CM_8 in the cumulative matrix with the larger of the two, E_4 or E_7.

CHAPTER 6

METHODOLOGY FOR ASSIGNING VALUES TO THE VARIATION TYPE AND TIME

6.1 Introduction to Monte Carlo

Monte Carlo simulation is a numerical method that accounts for risk in quantitative analysis and decision making. Named for the city in Monaco famed for its casinos and games of chance, the Monte Carlo method was invented in the 1940s by scientists as a numerical solution to complex physics problems such as neutron diffusion. Monte Carlo simulation uses random numbers as inputs to an inverse cumulative distribution function to simulate sampling from a population. These inputs are used to generate and explore an output that models complex business activities, plans and processes. Just like the physics problem of the 1940s, large scale project activities, plans and processes can also be modeled using Monte Carlo simulation.

6.2 Utilizing Monte Carlo on a Sequence of Events

For our illustration above, the timing of the variation conveniently occurred at the end of each event. In reality, the timing of the variation is unknown. Also, the type of variation that will occur is unknown. To examine the range of variation for each event, representative data is used to assign probabilities to the variation type and timing. Having these distributions, Monte Carlo analysis is performed to calculate the variation frequency for each event. See Appendix C for more Monte Carlo calculations.

For our illustration, the variation type distribution will be 80% contract variation, 40% quality variation, and 30% uncontrollable variation shown in figure 6.1 below.

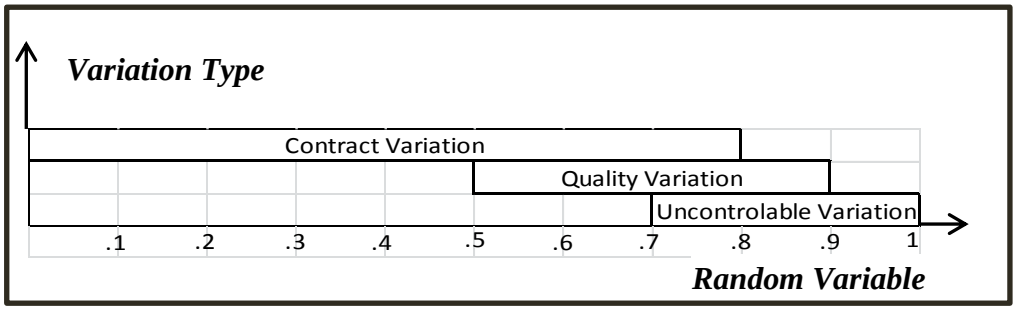


Figure 6.1. Variation Type Distribution

In the Monte Carlo simulation, N=4 variations, so 4 random numbers will be generated to designate the type of variation, and the distribution will fit the 80, 40, 30 distribution. For each of the 4 numbers, a second random number is generated to give the timing of the variation. For example, the first random number generated is used to determine the type of variation. If the first random number generated is between 0 and .5, then the type of variation is only a contract variation, and recall T_1=.5472, T_2=.2631, T_3=.1897, so .5472 is used in Eq. 3.7 to calculate F_1 = 1. If the random number is between .5 and .7, the types of variations are contract and quality variation, so .5472 and .2631 are used in Eq. 3.7 to calculate F_1 = .6753 and F_2=.3246. If the number generated is between .7 and .8, then all three types of variation are used in Eq. 3.7, forcing a value of 1 in the denominator, which results in F_1=.5472, F_2=.2631, F_3=.1897. If the random number is between .8 and .9, the types are quality and uncontrollable variation, so .2631 and .1897 are used in Eq. 3.7.

Consider Table 6.1 below.

Table 6.1. Variation Type and Timing

	N=4 variations							
	Variation 1		Variation 2		Variation 3		Variation 4	
Random Number->	Type 0.6	Time 1.5	Type 0.85	Time 4	Type 0.1	Time 5.5	Type 0.95	Time 8
	.5<.6>.7 so Variation 1 is a contract and quality variation	Variation 1 occurs at 1.5 weeks	.8<.85>.9 so Variation 3 is a Quality and uncontrollable variation	Variation 3 occurs at 4 weeks	0<.1>.5 so Variation 4 is a contract variation	Variation 4 occurs at 5.5 weeks	.9<.95>1 so Variation 2 is an uncontrollable variation	Variation 2 occurs at 8 weeks

To further illustrate, both the type and timing are shown in figures 6.2 and 6.3 below.

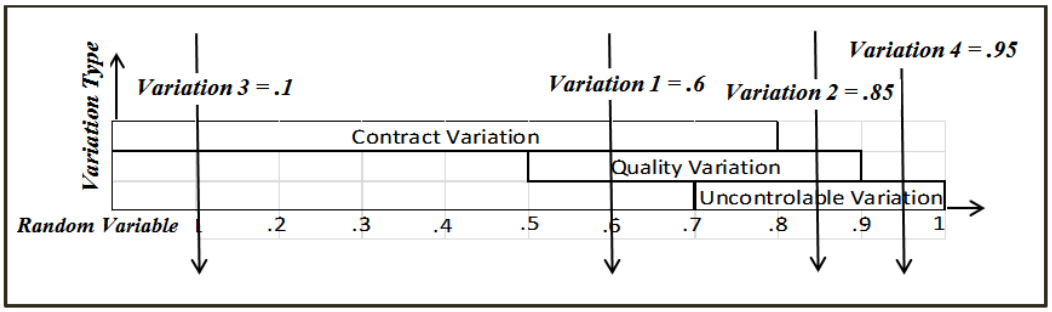


Figure 6.2. Variation Type

A probability distribution can also be used for the timing of each variation. The timing shown in figure 6.3 below is used for this example.

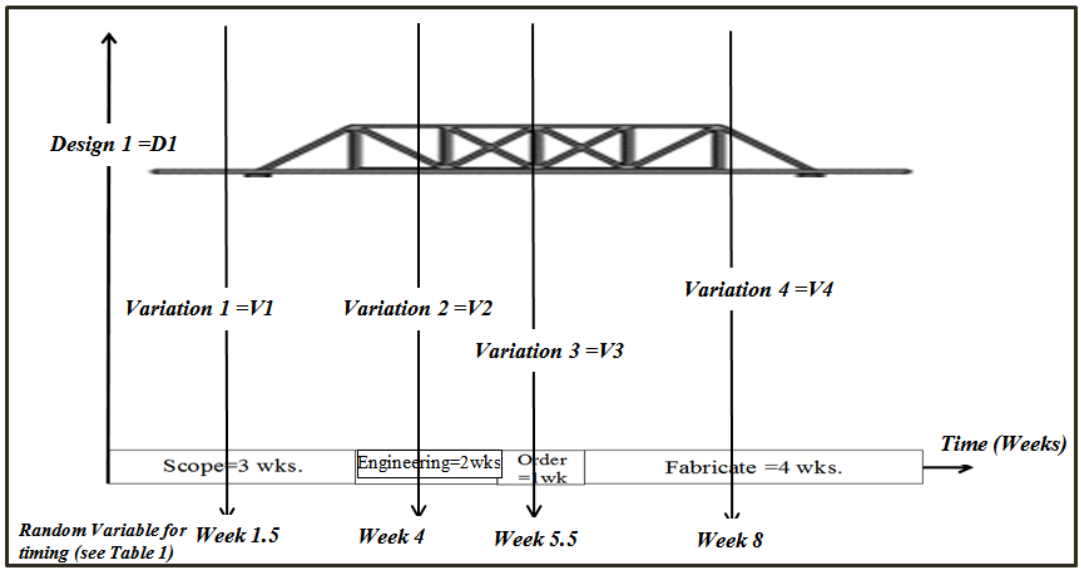


Figure 6.3. Variation Timing on the Bridge Illustration

Recall from the bridge illustration and our AHP analysis,

T_1 = .5472 , T2 = .2631 , T3 = .1897

Also, recall the results from the AHP for the four events with respect to each other within the types of variation yield:

$$[R]_1 = \begin{pmatrix} .392 \\ .162 \\ .054 \\ .392 \end{pmatrix}_1 \quad [R]_2 = \begin{pmatrix} .560 \\ .134 \\ .084 \\ .222 \end{pmatrix}_2 \quad \text{and} \quad [R]_3 = \begin{pmatrix} .484 \\ .086 \\ .102 \\ .328 \end{pmatrix}_3 .$$

Also, recall $[e_0] = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 4 \end{pmatrix}$.

The equations developed in sections 3.5 and 3.6 are used and listed below for convenience.

Also, $$\Delta_n = \sum_{k=1}^{K} \{ F_k \bullet M_k \} \circ [s]_n \qquad \text{Eq. 3.7}$$

Then, $$F_k = \frac{\mu_k}{\sum_{i=1}^{K} \mu_i T_i} T_k \qquad \text{Eq. 3.8}$$

$$M_k = \frac{1}{\sum_{p=1}^{P} \rho_p^k R_p^k} [\rho]_k \circ [R]_k \qquad \text{Eq. 3.9}$$

$$\text{Total } \Delta = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 \qquad \text{Eq.3.17}$$

Expanding Eq. 3.7 gives

$$\Delta_n = [\{ (F_1 \bullet M_1) + (F_2 \bullet M_2) + (F_3 \bullet M_3) \}] \circ [s]_n \qquad \text{Eq. 3.10}$$

where $$F_1 = \frac{\mu_1}{(\mu_1 \bullet T_1 + \mu_2 \bullet T_2 + \mu_3 \bullet T_3)} \bullet T_1 \qquad \text{Eq. 3.11}$$

$$F_2 = \frac{\mu_2}{(\mu_1 \bullet T_1 + \mu_2 \bullet T_2 + \mu_3 \bullet T_3)} \bullet T_2 \qquad \text{Eq. 3.12}$$

$$F_3 = \frac{\mu_3}{(\mu_1 \bullet T_1 + \mu_2 \bullet T_2 + \mu_3 \bullet T_3)} \bullet T_3 \qquad \text{Eq. 3.13}$$

and $$M_1 = \frac{1}{(\rho_1^1 \bullet R_1^1 + \rho_2^1 \bullet R_2^1 + \rho_3^1 \bullet R_3^1 + \rho_4^1 \bullet R_4^1)} \bullet [\rho]_1 \circ [R]_1 \qquad \text{Eq. 3.14}$$

$$M_2 = \frac{1}{(\rho_1^2 \bullet R_1^2 + \rho_2^2 \bullet R_2^2 + \rho_3^2 \bullet R_3^2 + \rho_4^2 \bullet R_4^2)} \bullet [\rho]_2 \circ [R]_2 \qquad \text{Eq. 3.15}$$

$$M_3 = \frac{1}{(\rho_1^3 \bullet R_1^3 + \rho_2^3 \bullet R_2^3 + \rho_3^3 \bullet R_3^3 + \rho_4^3 \bullet R_4^3)} \bullet [\rho]_3 \circ [R]_3 \qquad \text{Eq. 3.16}$$

$$[\rho]_1 = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{bmatrix}_1, \quad [\rho]_2 = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{bmatrix}_2 \quad \text{and} \quad [\rho]_3 = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{bmatrix}_3 .$$

$$[R]_1 = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}_1 \quad [R]_2 = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}_2 \quad \text{and} \quad [R]_3 = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}_3 .$$

For our illustration;

For Δ_1 there is a contract and quality component that occurs at week 1.5, so it only impacts event 1. Eq 3.10 is

$$\Delta_1 = [\{ (F_1 \bullet M_1) + (F_2 \bullet M_2) \}] \circ [s]_1 \qquad \text{Eq. 6.1}$$

where

$$F_1 = \frac{\mu_1}{(\mu_1 \bullet T_1 + \mu_2 \bullet T_2)} \bullet T_1 \qquad \text{Eq. 6.2}$$

$$F_1 = \frac{T_1}{(T_1 + T_2)} \qquad \text{Eq. 6.3}$$

and

$$F_2 = \frac{\mu_2}{(\mu_1 \bullet T_1 + \mu_2 \bullet T_2)} \bullet T_2 \qquad \text{Eq. 6.4}$$

$$F_2 = \frac{T_2}{(T_1 + T_2)} \qquad \text{Eq. 6.5}$$

and

$$M_1 = \frac{1}{(\rho_1^1 \bullet R_1^1 + 0 \bullet R_2^1 + 0 \bullet R_3^1 + 0 \bullet R_4^1)} \bullet \begin{pmatrix} \rho_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}_1 \circ [R]_1 \qquad \text{Eq. 6.6}$$

and

$$M_1 = \begin{pmatrix} 1/R_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}_1 \circ \begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{pmatrix}_1 \qquad \text{Eq. 6.7}$$

This results in

$$M_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}_1 \qquad \text{Eq. 6.8}$$

The same for M_2, which gives

$$M_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}_2 \qquad \text{Eq. 6.9}$$

$$\Delta_1 = \{ (\frac{T_1}{(T_1+T_2)} \bullet \begin{bmatrix}1\\0\\0\\0\end{bmatrix}_1) + (\frac{T_2}{(T_1+T_2)} \bullet \begin{bmatrix}1\\0\\0\\0\end{bmatrix}_2) \} \circ \begin{bmatrix}1.5\\0\\0\\0\end{bmatrix}_1 = \begin{bmatrix}1.5\\0\\0\\0\end{bmatrix}_1 \qquad \text{Eq. 6.10}$$

For Δ_2 there is an uncontrollable that occurs at week 8 so it impacts all events. Eq 3.10 is

$$\Delta_2 = [\{(F_3 \bullet M_3)\}] \circ [s]_2 \qquad \text{Eq. 6.11}$$

$$F3 = 1,\ M3 = \begin{bmatrix}R_1\\R_2\\R_3\\R_4\end{bmatrix}_3 \qquad \text{Eq. 6.12}$$

$$\Delta_2 = \{(1 \bullet \begin{bmatrix}.484\\.086\\.102\\.328\end{bmatrix}_3)\} \circ \begin{bmatrix}3\\2\\1\\2\end{bmatrix}_2 = \begin{bmatrix}1.452\\.172\\.102\\.656\end{bmatrix}_2 \qquad \text{Eq. 6.13}$$

For Δ_3 there is a quality and uncontrollable that occurs at week 4, so it impacts two events. Eq 3.10 is

$$\Delta_3 = [\{ (F_2 \bullet M_2) + (F_3 \bullet M_3) \}] \circ [s]_3 \qquad \text{Eq. 6.14}$$

where

$$F_2 = \frac{\mu_2}{(\mu_2 \bullet T_2 + \mu_3 \bullet T_3)} \bullet T_2 \qquad \text{Eq. 6.15}$$

$$F_2 = \frac{T_2}{(T_2+T_3)} \qquad \text{Eq. 6.16}$$

and

$$F_3 = \frac{\mu_3}{(\mu_2 \bullet T_2 + \mu_3 \bullet T_3)} \bullet T_3 \qquad \text{Eq. 6.17}$$

$$F_3 = \frac{T_3}{(T_2+T_3)} \qquad \text{Eq. 6.18}$$

and

$$M_2 = \frac{1}{(\rho_1^1 \bullet R_1^1 + \rho_2^1 \bullet R_2^1 + 0 \bullet R_3^1 + 0 \bullet R_4^1)} \bullet \begin{bmatrix}\rho_1\\\rho_2\\0\\0\end{bmatrix}_2 \circ [R]_2 \qquad \text{Eq. 6.19}$$

and

$$M_2 = \begin{pmatrix}1/(R_1 + R_2)\\1/(R_1 + R_2)\\0\\0\end{pmatrix}_2 \circ \begin{bmatrix}R_1\\R_2\\R_3\\R_4\end{bmatrix}_2 = \begin{pmatrix}R_1/(R_1 + R_2)\\R_2/(R_1 + R_2)\\0\\0\end{pmatrix}_2 \qquad \text{Eq. 6.20}$$

Similarly,

$$M_3 = \begin{pmatrix} R_1/(R_1 + R_2) \\ R_2/(R_1 + R_2) \\ 0 \\ 0 \end{pmatrix}_2 \qquad \text{Eq. 6.21}$$

$$\Delta_3 = \{(\frac{T_2}{(T_2+T_3)} \bullet \begin{pmatrix} R_1/(R_1 + R_2) \\ R_2/(R_1 + R_2) \\ 0 \\ 0 \end{pmatrix}_2) + (\frac{T_3}{(T_2+T_3)} \bullet \begin{pmatrix} R_1/(R_1 + R_2) \\ R_2/(R_1 + R_2) \\ 0 \\ 0 \end{pmatrix}_3) \} \circ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}_3 \qquad \text{Eq. 6.22}$$

$$\frac{T_2}{(T_2+T_3)} = .2631/(.2631+.1897) = .581, \ \frac{T_3}{(T_2+T_3)} = .1897/(.2631+.1897) = .419 \qquad \text{Eq. 6.23}$$

For event quality ranking,

$R_1/(R_1+R_2) = .560/(.560 + .134) = .807$

$R_1/(R_1+R_2) = .134/(.560 + .134) = .193$

For event uncontrollable ranking,

$R_1/(R_1+R_2) = .484/(.484 + .086) = .849$

$R_1/(R_1+R_2) = .086/(.484 + .086) = .151$

$$\Delta_3 = \{(.581 \bullet \begin{pmatrix} .807 \\ .193 \\ 0 \\ 0 \end{pmatrix}_2) + (.419 \bullet \begin{pmatrix} .849 \\ .151 \\ 0 \\ 0 \end{pmatrix}_3) \} \circ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}_3 = \begin{pmatrix} 2.47 \\ 0.18 \\ 0 \\ 0 \end{pmatrix}_3 \qquad \text{Eq. 6.24}$$

For Δ_4 there is a contract that occurs at week 5.5, so it impacts three events. Eq 10 is

$$\Delta_4 = [\{(F_1 \bullet M_1)\}] \circ [s]_4 \qquad \text{Eq. 6.25}$$

F1 = 1,

and $$M_1 = \frac{1}{(\rho_1^1 \bullet R_1^1 + \rho_2^1 \bullet R_2^1 + \rho_3^1 \bullet R_3^1 + 0 \bullet R_4^1)} \bullet \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ 0 \end{bmatrix}_2 \circ [R]_1 \qquad \text{Eq. 6.26}$$

and $$M_1 = \begin{pmatrix} 1/(R_1 + R_2 + R_3) \\ 1/(R_1 + R_2 + R_3) \\ 1/(R_1 + R_2 + R_3) \\ 0 \end{pmatrix}_1 \circ \begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{pmatrix}_1 = \begin{pmatrix} R_1/(R_1 + R_2 + R_3) \\ R_2/(R_1 + R_2 + R_3) \\ R_3/(R_1 + R_2 + R_3) \\ 0 \end{pmatrix}_1 \qquad \text{Eq. 6.27}$$

$$\Delta_4 = \{(1 \bullet \begin{pmatrix} R_1/(R_1 + R_2 + R_3) \\ R_2/(R_1 + R_2 + R_3) \\ R_3/(R_1 + R_2 + R_3) \\ 0 \end{pmatrix}_1)\} \circ \begin{pmatrix} 3 \\ 2 \\ .5 \\ 0 \end{pmatrix}_4 \qquad \text{Eq. 6.28}$$

$$R_1/(R_1 + R_2 + R_3) = .392/(.392+.162+.054) = .64 \qquad \text{Eq. 6.29}$$

$$R_2/(R_1 + R_2 + R_3) = .162/(.392+.162+.054) = .27 \qquad \text{Eq. 6.30}$$

$$R_3/(R_1 + R_2 + R_3) = .054/(.392+.162+.054) = .09 \qquad \text{Eq. 6.31}$$

$$\Delta_4 = \begin{pmatrix} .64 \\ .27 \\ .09 \\ 0 \end{pmatrix}_4 \circ \begin{pmatrix} 3 \\ 2 \\ .5 \\ 0 \end{pmatrix}_4 = \begin{pmatrix} 1.93 \\ 0.53 \\ 0.04 \\ 0 \end{pmatrix}_4 \qquad \text{Eq. 6.32}$$

$$\text{Total } \Delta = \sum_{n=1}^{N} \Delta_n \qquad \text{Eq. 3.6}$$

$$\text{Total } \Delta = \begin{pmatrix} 1.5 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1.452 \\ .172 \\ .102 \\ .656 \end{pmatrix} + \begin{pmatrix} 2.474 \\ .175 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1.934 \\ .533 \\ .044 \\ 0 \end{pmatrix} = \begin{pmatrix} 7.36 \\ .88 \\ .146 \\ .656 \end{pmatrix} \qquad \text{Eq. 6.33}$$

This process is repeated for 1000 iterations to give a probability distribution in figures 6.4, 6.5, 6.6, and 6.7 shown below.

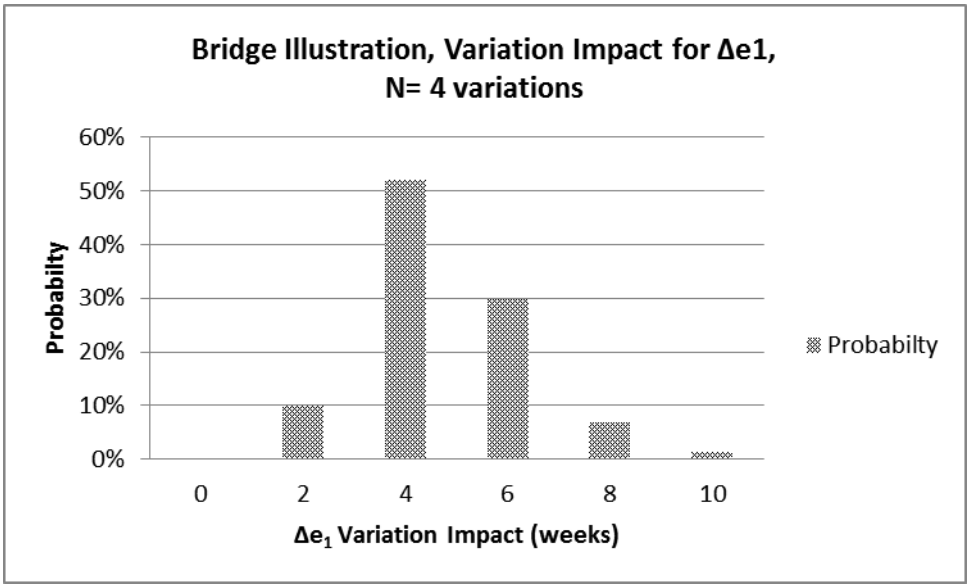


Figure 6.4. Variation Impact for Four Variations (N=4), on Event 1 (e_1)

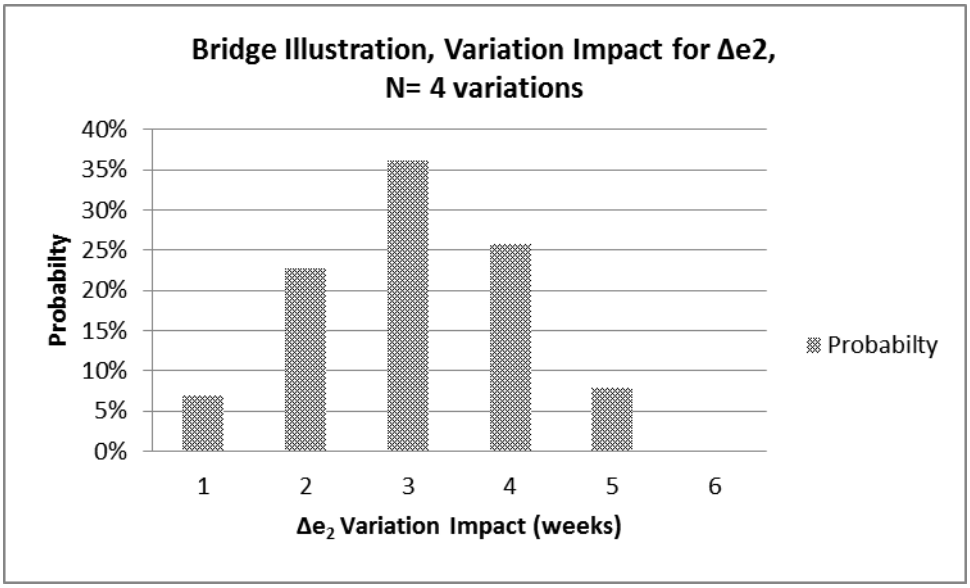


Figure 6.5. Variation Impact for Four Variations (N=4), on Event 2 (e_2)

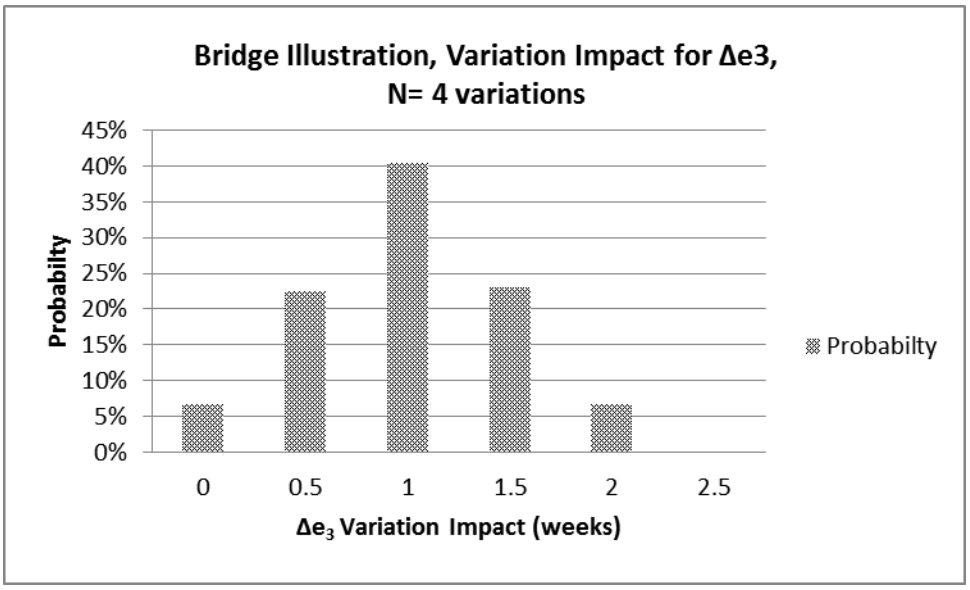


Figure 6.6. Variation Impact for Four Variations (N=4), on Event 3 (e_3)

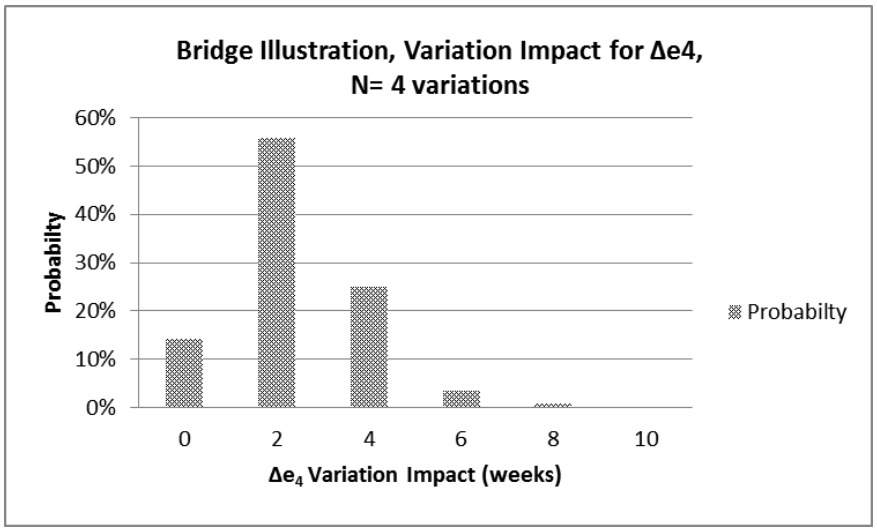


Figure 6.7. Variation Impact for Four Variations (N=4), on Event 4 (e_4)

Notice that the highest probability is adding 4 weeks to event 1, 3 weeks to event 2, 1 week to event 3 and 2 weeks to event 4. To calculate the impact of all 4 variations, equation 3.7 is used to calculate each Δ_n ; then equation 3.6 is used to find the total Δ. The total Δ is used in equation 3.64 to calculate $[e]_n$; then equations 3.1b and 3.2 are used in Equation 3.66 to calculate

the total project time increase. For the total impact, as indicated in figure 6.8, the highest probability is 70% of adding 10 weeks to the completion time if there are 4 variations that occur during project progression.

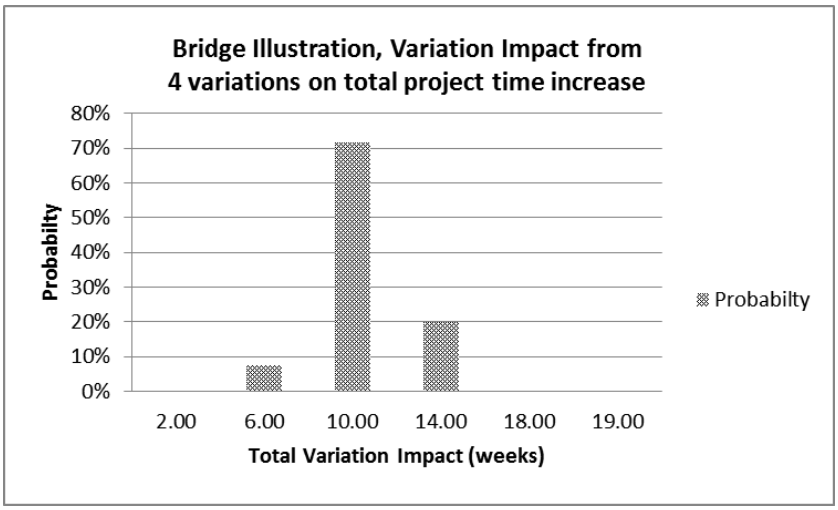


Figure 6.8. Total Variation Impact (N=4), on All Events (e_1+e_2+e_3+e_4)

The cumulative density curve in figure 6.9 shows a 100% probability at 14 weeks.

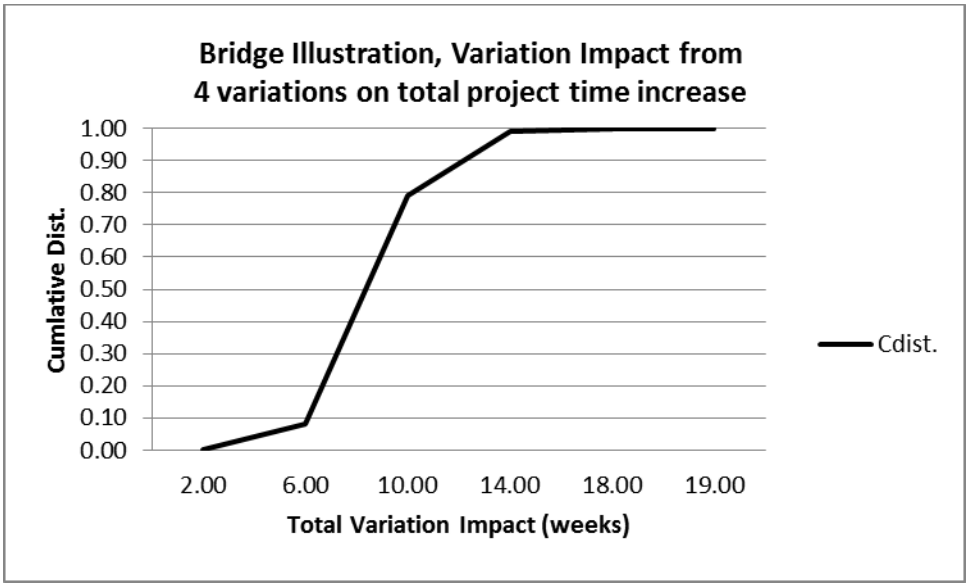


Figure 6.9. Cumulative Distribution for the Total Variation Impact

6.3 Results

Note that the variation impact decreases going from the first event to the last event. This decrease is due to the timing of the variation. Every variation will impact the first event because every event occurs after event one is partially or fully completed. Also note that the model used in this illustration results in a 100% probability that the variation impact will be 14 weeks or less (see figure 6.9). The initial project time is 10 weeks, so there is 100% probability that the project will be 140% longer than the initial planned timeline if there are four variations during project progression.

Examining the variation impact of each event separately is valuable for exposing which event has the most opportunity for decreasing the total variation impact. For example, the frequency charts show that event one has a 51% probability of 4 weeks and a 29% probability of 6 weeks while event two has a 24% probability of 2 weeks, a 36% probability of 3 weeks, and a 27% probability of 4 weeks. Of the four event frequency charts, the ones for event one and event two show that these events have the most opportunity for decreasing the impact of variation.

6.4 Utilizing Monte Carlo on Two Parallel Sequence of Events

Figure 6.10 shows two parallel coupled sequential events for the pump illustration.

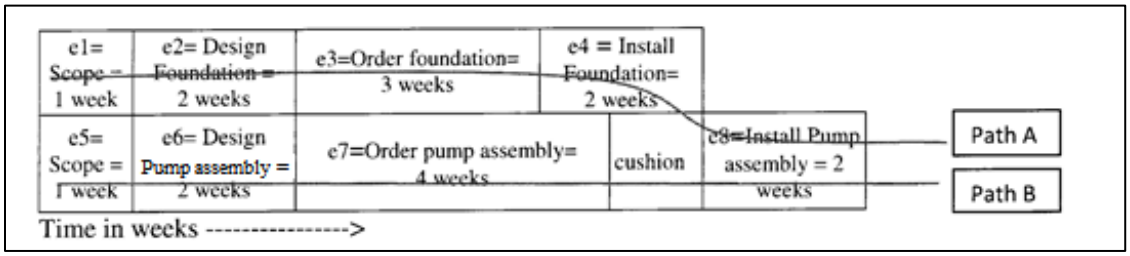


Figure 6.10. Two Parallel Coupled Sequential Events

The precedence matrix, where e4 must be completed before e8, for these two coupled sequence of events is:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 3 \\ 6 \\ 0 \\ 1 \\ 3 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \\ 1 \\ 3 \\ 7 \\ 10 \end{bmatrix}$$

PM [e] + [CM] = [E]

In the illustration above, it is reasonable to examine the condition that the foundation must be installed (e4) before the pump can be installed (e8). This condition will be referred to as the one utilizing precedent matrix A shown below.

Precedence Matrix A

0	0	0	0	0	0	0	0	1	0	1
1	0	0	0	0	0	0	0	2	1	3
0	1	0	0	0	0	0	0	3	3	6
0	0	1	0	0	0	0	0	2	6	8
0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	1	0	0	0	2	1	3
0	0	0	0	0	1	0	0	4	3	7
0	0	0	1	0	0	1	0	2	8	10

Invoking another constraint on the project, ordering the foundation (e3) must be completed before ordering the pump assembly (e7). This condition will be referred to as the one utilizing precedent matrix B shown below.

Precedence Matrix B

0	0	0	0	0	0	0	0	1	0	1
1	0	0	0	0	0	0	0	2	1	3
0	1	0	0	0	0	0	0	3	3	6
0	0	1	0	0	0	0	0	2	6	8
0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	1	0	0	0	2	1	3
0	0	1	0	0	1	0	0	4	6	10
0	0	0	1	0	0	1	0	2	10	12

Examining the Monte Carlo results for one variation from, adding the constraint increases the variation impact illustrated in figure 6.11 below. The constraint in precedent matrix A gives 35% probability of adding 3 weeks and 38% probability of adding 4.5 weeks to the project completion time if there are four variations during project progression. The constraint in precedent matrix B gives 25% probability of adding 1.5 and 6 weeks to the project completion time if there is one variation during project progression.

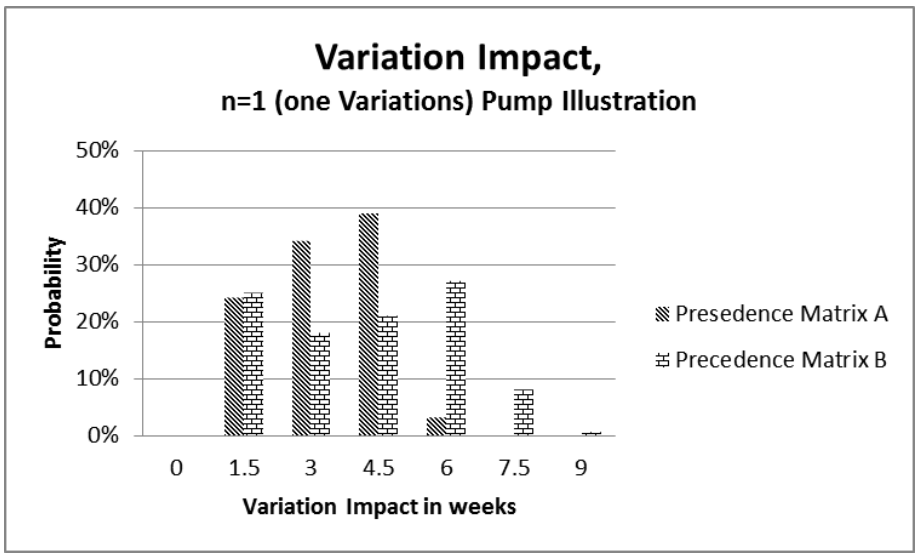


Figure 6.11. Variation Impact for One Variation on Precedence Matrix A vs. B

The cumulative density results in figure 6.12 show 100% probability shifting from 6 weeks to 7.5 weeks added to completion time when comparing precedent matrix A to matrix B for one variation.

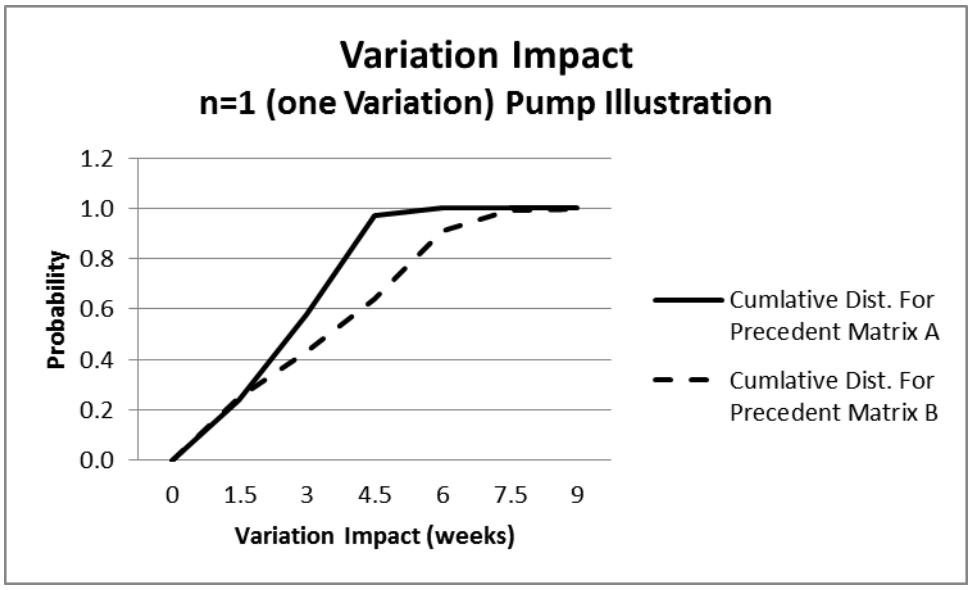


Figure 6.12. Cumulative Distribution of the Variation Impact for One Variation on Precedence Matrix A vs. B

Examining the Monte Carlo results for two variations, adding the constraint increases the variation impact illustrated in figures 6.13 and 6.14 below. The constraint in precedent matrix A gives 35% probability of adding 4.5 weeks and 45% probability of adding 6.75 weeks to the project completion time if there are four variations during project progression. The constraint in precedent matrix B gives 30% probability of adding 6.75 weeks and 25% probability of adding 9 weeks to the project completion time if there are two variations during project progression.

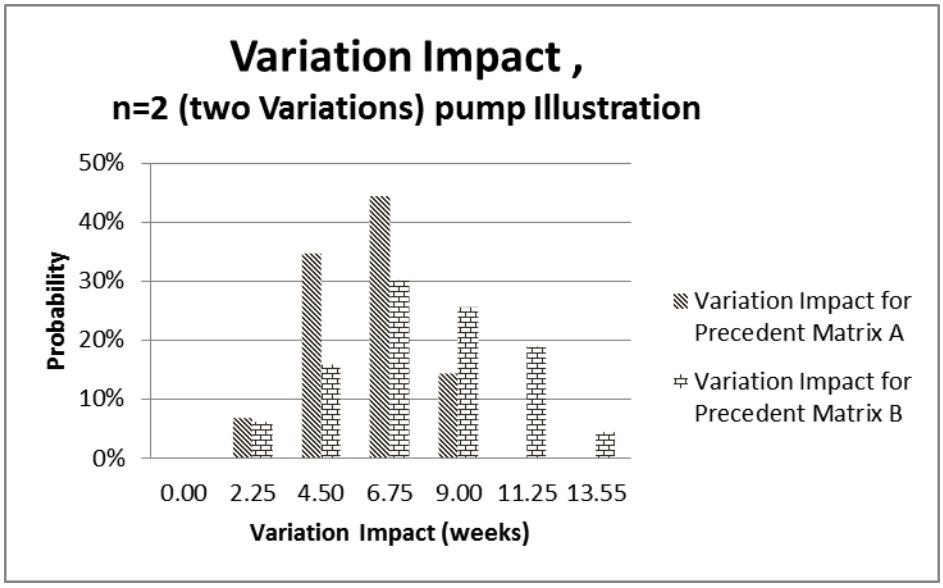


Figure 6.13. Variation Impact for Two Variations on Precedence Matrix A vs. B

The cumulative density results show 100% probability shifting from 9 weeks to 13.55 weeks added to completion time when comparing precedent matrix A to matrix B for two variations.

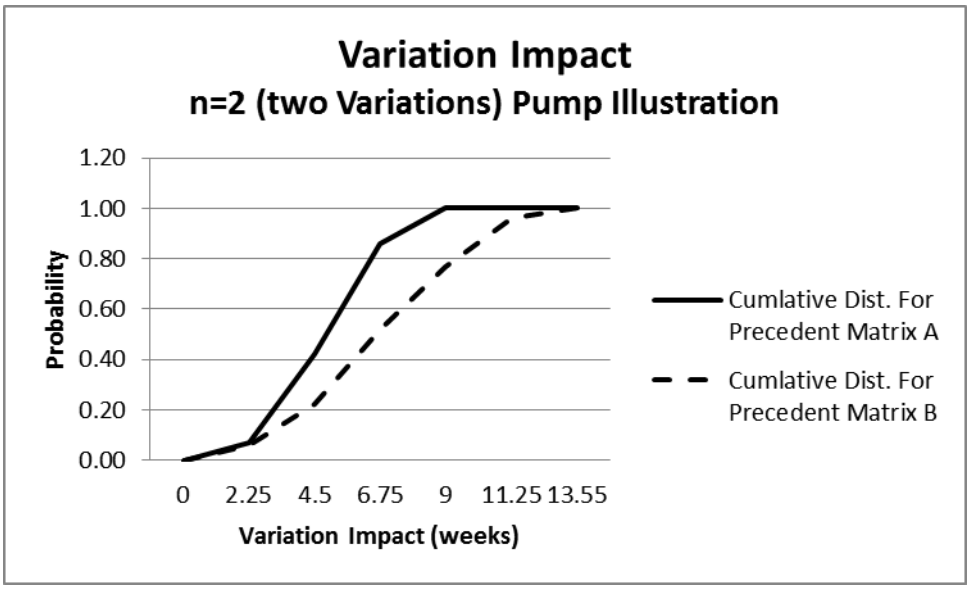


Figure 6.14. Cumulative Distribution of the Variation Impact for Two Variations on Precedence Matrix A vs. B

Examining the Monte Carlo results for three variations, adding the constraint increases the variation impact as illustrated in figures 6.15 and 6.16 below. The constraint in precedent matrix A gives 33% probability of adding 6.8 weeks and 51% probability of adding 3.2 weeks to the project completion time if there are four variations during project progression. The constraint in precedent matrix B gives 33% probability of adding 10.2 weeks and 13.6 weeks to the project completion time if there are three variations during project progression.

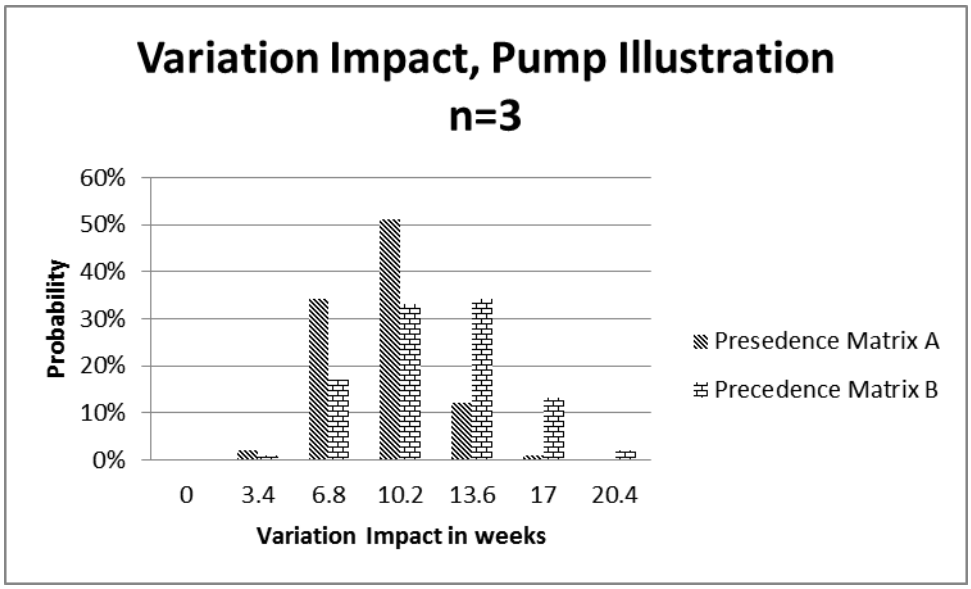


Figure 6.15. Variation Impact for Three Variations on Precedence Matrix A vs. B

The cumulative density results show 100% probability shifting from 13.6 weeks to 17 weeks added to completion time when comparing precedent matrix A to matrix B for 3 variations.

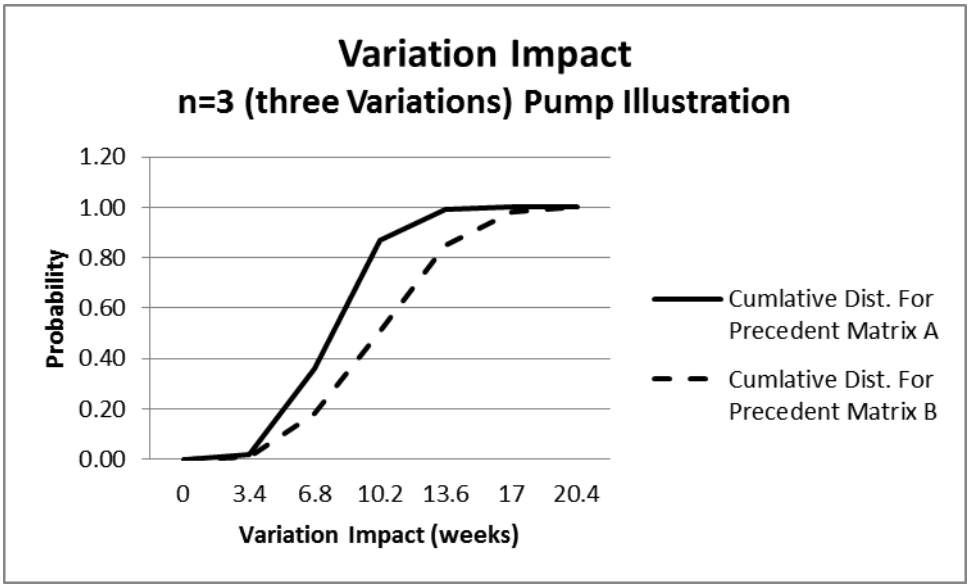


Figure 6.16. Cumulative Distribution of the Variation Impact for Three Variations on Precedence Matrix A vs. B

Examining the Monte Carlo results for four variations, adding the constraint increases the variation impact illustrated in figures 6.17 and 6.18 below. The constraint in precedent matrix A gives 30% probability of adding 9 weeks and 60% probability of adding 13.5 weeks to the project completion time if there are four variations during project progression. The constraint in precedent matrix B gives 38% probability of adding 13.5 weeks and 37% probability of adding 18 weeks to the project completion time if there are four variations during project progression.

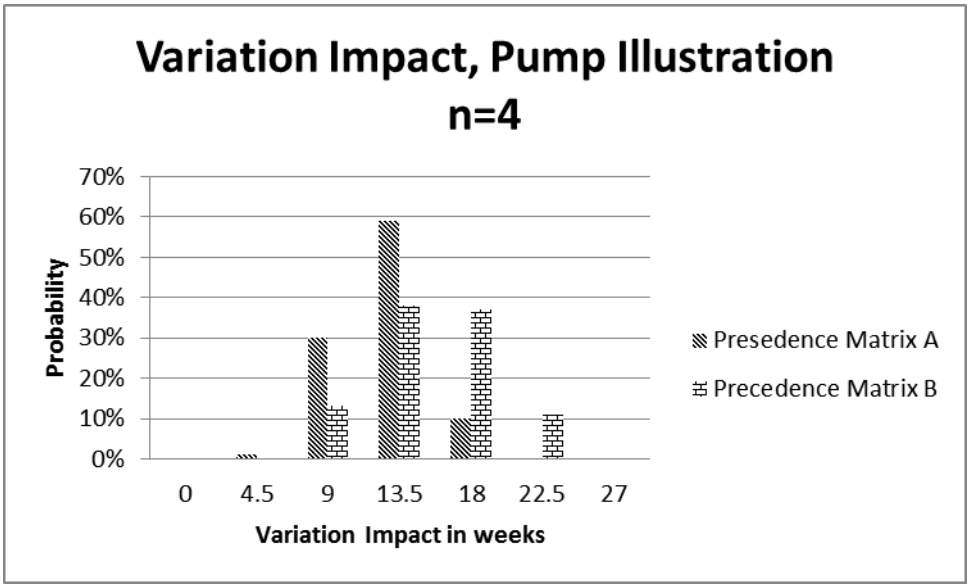


Figure 6.17. Variation Impact for Four Variations on Precedence Matrix A vs. B

The cumulative density results show 100% probability shifting from 18 weeks to 22.5 weeks added to completion time when comparing precedent matrix A to matrix B for four variations.

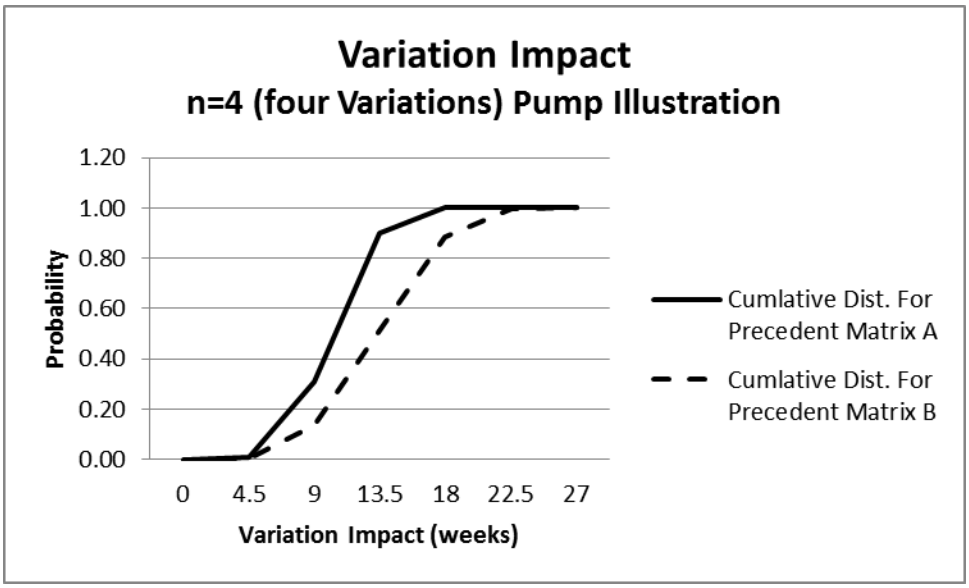


Figure 6.18. Cumulative Distribution of the Variation Impact for Four Variations on Precedence Matrix A vs. B

6.5 Results

Figures 6.12, 6.14, 6.16, and 6.18 illustrate how the added constraint increases the probability of a higher variation impact. Intuitively, these results are consistent with the expectation of having a longer completion time for projects with an increasingly rigid plan of events. One benefit of quantifying the probability of numerous scenarios is that the figures can be used to manage performance and cost issues. Another positive attribute of having a magnitude assigned to the variation impact, in this case the number of weeks the project will be extended, is that it allows contingency plans to be made for mitigating or avoiding unnecessary consequences as a result of the variation or unplanned event.

6.6 More Complex Situations

The bridge and pump illustrations are simple projects used to explain the fundamental equations and concepts for calculating the impact variation has on a project. Figure 6.19 shows how the methods and formulas derived are general and flexible enough to be used on any number of parallel coupled sequences of events needed to complete a large scale project. For example, if a project has 10 different variation types and 1000 parallel sequences, each having 1000 events, for each variation, equation 3.1b, derived earlier,

$$[e]_n + [CM]_n = [E]_n \qquad \text{Eq. 3.1b}$$

can be used with equation 3.7, where K= 10 and the column vector [e] will have 1000 events.

$$\Delta_n = \sum_{k=1}^{K} [\{ F_k \bullet M_k \}] \circ [s]_n \qquad \text{Eq. 3.7}$$

The binomial μ_k is used to determine which types belong to each variation in equation 3.8 where T_k is the relative ranking of the 10 types of variation.

$$F_k = \frac{\mu_k}{\sum_{i=1}^{K} \mu_i T_i} T_k \qquad \text{Eq. 3.8}$$

and ρ_p is used to determine if event p belongs to the variation in equation 3.9, where p will go from 1 to 1000.

$$M_k = \frac{1}{\sum_{p=1}^{P} \rho_p^k R_p^k} \; [\rho]_k \circ [R]_k \qquad \text{Eq. 3.9}$$

The precedent matrix would be a 1000 x 1000 element matrix defining the constraints each sequence of events has with one another. Then, substituting the $[e]_n$ from Eq. 3.64 into Eq. 3.2 and equation 3.66 gives the change in completion time

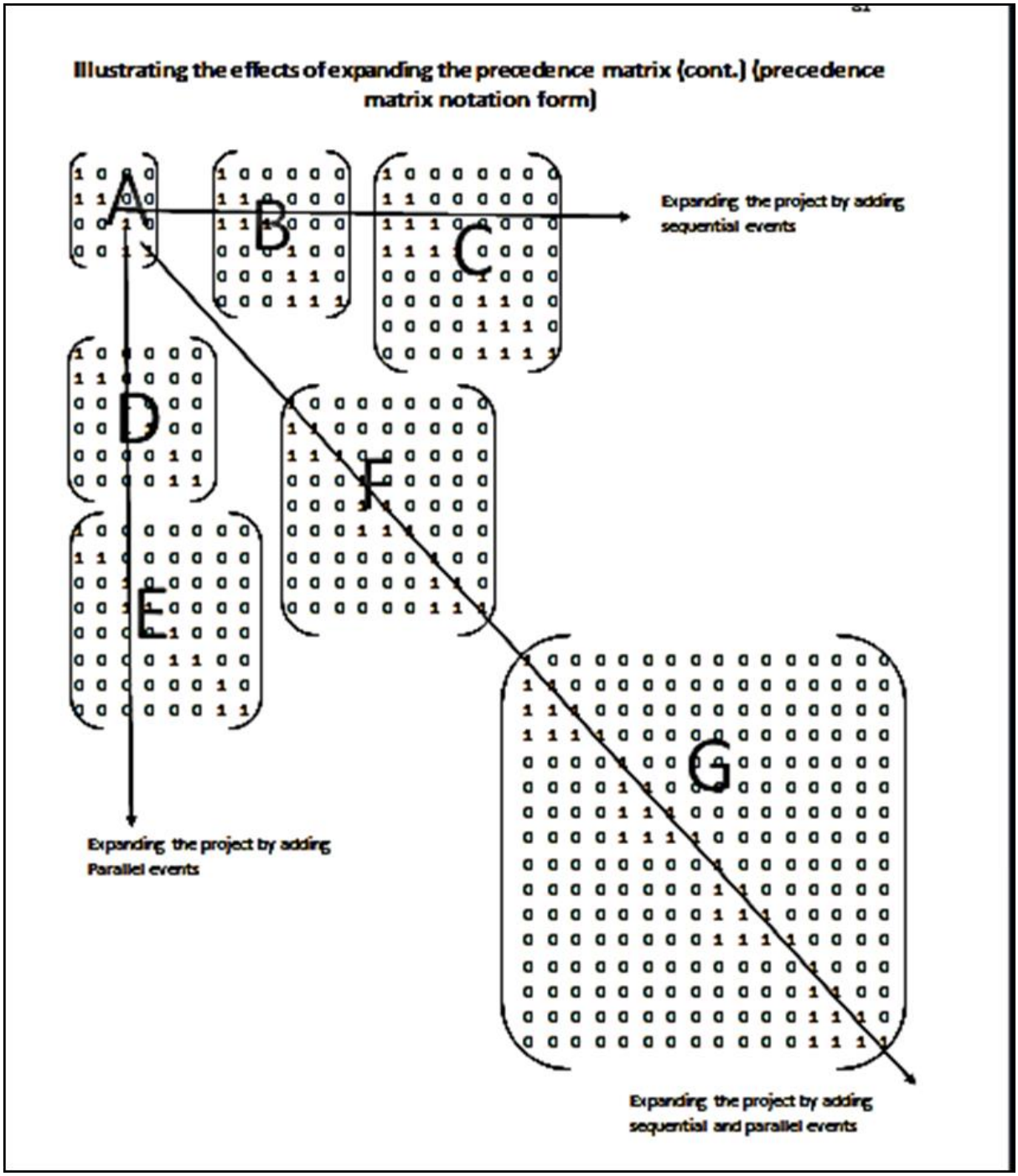


Figure 6.19. Expanding the Precedence Matrix

CHAPTER 7
CONCLUSION

Managing projects consists of a complex and integrated array of decisions, actions, and communications necessary to complete projects successfully. Identifying and quantifying the relationships between project changes and an initial project plan is vitally important to a large scale project. Project changes continue to challenge areas such as accounting, finance, planning, and production. This dissertation formulates and illustrates a method for quantifying the impact that variation has on the progression of a large project. It first formulates a way to calculate the variation impact on a single series of events. It further expounds the formula by deriving a general expression for calculating the dynamic impact of variation for a series of coupled parallel events. The general equation used to annotate this method can be fitted to a wide variety of situations.

One key aspect of this proposed methodology is its ability to conform to a wide range of applications by using types of variations within each variation allowing it to conform to the actual project situation. The resulting formula can have as many variations as are expected to occur during project progression. The equation uses a convention for assigning multipliers to adjust an initial root timeline accounting for the type of variation and the specific influence the variation has on each event. The formula is used with a set of representative data to show that variation mostly impacts events that are partially or fully completed. Of the many options available, AHP is used to illustrate one method of assigning values to the multipliers. A precedence matrix is used to couple a series of events using constraints. This technique provides a method for representing a large project timeline. Monte Carlo methods are used to generate frequency plots and probability plots to show the likelihood of different impacts. The results illustrated in cumulative density

curves show how the added constraint increases the probability of a higher variation impact. These results are consistent with the expectation of having a longer completion time for projects with an increasingly rigid plan of events. Quantifying the probability of numerous scenarios provides a tool for managing performance and cost issues. The proposed method also assigns a magnitude to the variation impact allowing contingency plans to be made to avoid unnecessary consequences caused by the variation or unplanned event.

REFERENCES

Alkass, S., Mazerolle, M., and Harris, F. (1996). “Construction delay analysis techniques.” Journal of Construction Management and Economics, 14, 375-394.

Belton, V. and Gear, T. (1983). On a Short-coming of Saaty's Method of Analytic Hierarchies. Omega, 228-230.

Bordoli, D. W. and Baldwin, A. A. (1998). “A methodology for assessing construction project delays.” Journal of Construction Management and Economics, 16, 327-337.

Brown, D. W. (1988). *Influences on cost of construction changes and forward change pricing report.* Constr. Education Inst., the Frisby Group, Greenville, S.C.

Building Research Advisory Board (1978), *Exploratory Study on Responsibility, Liability and Accountability for Risks in Construction,* National Academy of Sciences, Washington, D.C..

Construction Industry Institute. (1988). "The impact of changes on construction cost and schedt*." *Rep. of CostISchedule Task Force,* Austin, Tex.

Dellon, A. L., and Dellon, (1988). "Documentation and verification in **the** change order process." *Trans.,* Am. Assoc. of Cost Engrs., C.7. I-C.7.5.El-Haik, B., & Al-Aomar, R. (2006). *Simulation-based Lean Six-Sigma and Design for Six-Sigma*: Wiley.

Dyer, J.S., and Wendell, R.E. (1985). A Critique of the Analytic Hierarchy Process, Working Paper, 84/85-4-24, Department of Management, The University of Texas at Austin, Austin, TX.

El-Haik, B., and Al-Aomar, Raid., (2006). “Simulation-Based Lean Six-Sigma and Design for Six-Sigma.” Published by John Wiley & Sons, Inc, 134-135.

Finke, M. R. (1999). “Window analysis of compensable delays.” Journal of Construction Engineering and Management, 125(2), 96-100.

Forman, Ernest H., "Ideal and Distributed Synthesis Modes for the Analytic Hierarchy Process" presented at the International Federation of Operations Research, Lisbon Portugal, July 1993.

Forman, Ernest H.; Saul I. Gass (July 2001). "The analytical hierarchy process—an exposition". *Operations Research* 49 (4): 469–487. doi:10.1287/opre.49.4.469.11231.

Galloway, P. D. and Nielsen, K.R. (1990). “Concurrent schedule delay in international contracts.” The International Construction Law Review. Pt.4, 386-401.

Hamilton, C., Stiller, A, and Sullivan P. E.; Statement before the Subcommittee on Projection Forces of the House Armed Services Committee on Shipbuilding; April 5, 2006; cited at

http://www.globalsecurity.org/military/library/congress/2006_hr/060405-sullivan-stiller-hamilton.pdf Procurement Cost Drivers

Hasse, M. (1961). Über die Behandlung graphentheoretischer Probleme unter Verwendung der Matrizenrechnung. *Wiss. Zeit. Tech. Univ. Dresden* 10, 1313–6

Hegazy, T. and Zhang (2005). “Daily Window Delay Analysis.” Journal of Construction Engineering and Management, 131(5), 505-512.

Hester, W. T., Kuprenas, **J.** A., and Chang, T. C. (1988). "Changes during construction and their impact: a review of the literature and development of management techniques." *Draft Rep.',* to Constr. Industry Inst., Austin, Tex., 26.Friday, 14 January (2011)

Kumaraswamy, M. M. and Yogeswaran, K. (2003). “Substantiation and assessment of laims for extensions of time.” International Journal of Project Management, 21(1),7-38.

Landau, E. (1914). *" Über Preisverteilung bei Spielturnieren*. *Zeitschrift für Mathematik und Physik*, 63 band (1914), p. 192

Leonard, C. A. (1987). "The effect of change orders on productivity." (1987). *Revay Rep.*, Revay and Associates, Ltd., Montreal, Canada, 6(2), 1-3.

Lovejoy, V. A. (2004). “Claims schedule development and analysis: Collapsed as built scheduling for beginners.” Journal of Cost Engineering, 46(1), 27-30.

Lucas, D. E. (2002). “Schedule Analyser Pro – An aid in the analysis of delay time impact analysis.” Journal of Cost Engineering, 44(8), 30-36.

McCaffrey, James (June 2005). "Test Run: The Analytic Hierarchy Process". *MSDN Magazine*. Retrieved 2007-08-21.

Mather, H., & Management, Institute of Operations. (1999). *Competitive Manufacturing*: CRC PressINC.

Miller, G.A. (1956). The Magical Number Seven Plus or Minus Two: Some Limits on Our Capacity for Processing Information. Psychological Review, 63: 81-97.

Ndekugri I, Braimah N and Gameson R- 2008 Delay Analysis within Construction Contracting Organizations http://bura.brunel.ac.uk/bitstream/2438/8289/2/Fulltext.pdf 6-10.

Nelson, Derek, (2011), "The Analysis and valuation of disruption", 25, Jan 2011 Hill International

Pickavance, K. (2005). Delay and disruption in Construction Contracts, 3rd Ed., LLP Reference Publishing, London.

Ramanujacharyulu, C (1964). "Analyses of preferential experiments". *Psychometrika* 29 (3): 257–261. doi:10.1007/bf02289722.

Saaty, Rozann, ed. (2009). "Participant Names and Papers". *Proceedings of the 10th International Symposium on the Analytic Hierarchy/Network Process*. Pittsburgh, Pennsylvania: ISAHP

Saaty, Thomas L. (2001). *Fundamentals of Decision Making and Priority Theory*. Pittsburgh, Pennsylvania: RWS Publications. ISBN 0-9620317-6-3.

Saaty, T.L. (1980). The Analytic Hierarchy Process. McGraw-Hill International, New York, NY, U.S.A.

Saaty, T.L. (1994). Fundamentals of Decision Making and Priority Theory with the AHP. RWS Publications, Pittsburgh, PA, U.S.A.

Sterman, John D. (2000). Business Dynamics - Systems Thinking and Modeling for a Complex World, Irwin McGraw-Hill.

Triantaphyllou, E., (1993). A Quadratic Programming Approach in Estimating Similarity Relations. IEEE Transactions on Fuzzy Systems, 1: 138-145.

Triantaphyllou, E., Lootsma, F.A., Pardalos, P.M., and Mann, S.H. (1994). On the Evaluation and Application of Different Scales For Quantifying Pairwise Comparisons in Fuzzy Sets. Multi-Criteria Decision Analysis, 3: 1-23.

Wickwire, J. M., Hurlbut, S. B. and Lerman, L. J. (1989). “Use of Critical path method techniques in contract claims: Issues and development, 1974 to 1988.” Public Contract Law Journal, 18, 338-391.

Wickwire, J. M. and Groff, M. J. (2004). Update on CPM proof of delay claims. Schedule Update-Project Management Institute College of Scheduling, 1(3), 3-9.

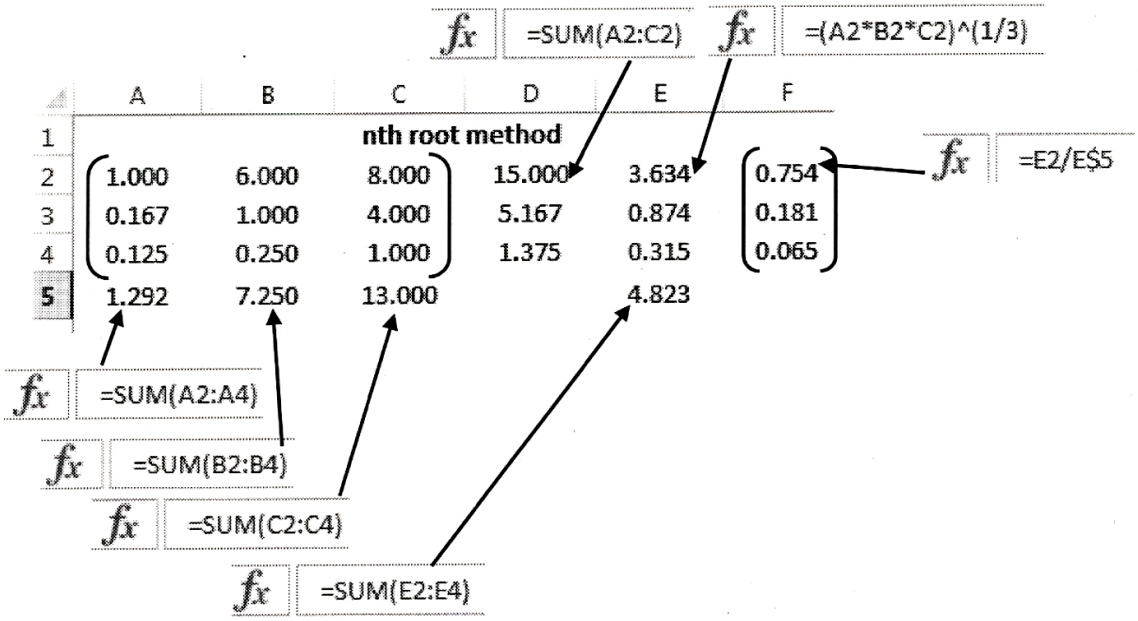
Zermelo, E. (1928). *Die Berechnung der Turnier-Ergebnisse als ein Maximumproblem der Wahrscheinlichkeitsrechnung*, *Mathematische Zeitschrift* 29, 1929, S. 436–460

Bramble, Barry B. and M. T. Callahan. (2000). *Construction Delay Claims*. Aspen Publishers.

APPENDIX A

Solving for the Pairwise Matrices Right Principal Eigenvector

Saaty proclaims that the relative ranking of the choice options set up in the pairwise matrix is found in the estimate of the right principal eigenvector of the pairwise matrix. The maximum left eigenvector of the pairwise matrix is estimated by using the geometric mean of each row. The illustration below shows how the elements in each row are multiplied with each other and then the n-th root is taken where n is the size of the matrix. The results are numbers are normalized to give the desired results.



Another way of solving for the Eigen vector is:

Solving for the Eigen values using the normalized sum of each row, then squaring the matrix and normalize the sum of the rows of the squared matrix. Then check the variation of the previous estimation and stop iterations if the variation is within tolerance. The example below illustrates the method described above. The iteration stops at 3 because the difference between the second and third is within tolerance.

	A	B	C	D	E
7					
8			Initial calculation		
9	1.000	6.000	8.000	15.000	0.696
10	0.167	1.000	4.000	5.167	0.240
11	0.125	0.250	1.000	1.375	0.064
12				21.542	

fx =SUM(A9:C9)

fx =D9/D$12

fx =D10/D$12

fx =D11/D$12

fx =SUM(A10:C10)

fx =SUM(A11:C11)

fx =SUM(D9:D11)

fx {=MMULT(A9:C11,A9:C11)}

	A	B	C	D	E
14			First Iteration		
15	3.000	14.000	40.000	57.000	0.763
16	0.833	3.000	9.333	13.167	0.176
17	0.292	1.250	3.000	4.542	0.061
18				74.708	
19			Second Iteration		
20	32.333	134.000	370.667	537.000	0.753
21	7.722	32.333	89.333	129.389	0.181
22	2.792	11.583	32.333	46.708	0.066
23				713.097	
24			Third Iteration		
25	3115.000	12958.889	35940.444	52014.333	0.754
26	748.759	3115.000	8639.259	12503.019	0.181
27	269.977	1123.139	3115.000	4508.116	0.065
28				69025.468	

The difference in Iterations are within tolerance.

Programs such as Mat Lab can also be used to solve for Eigen vector as shown below.

```
A =

    1.0000    3.0000    2.0000
    0.3333    1.0000    2.0000
    0.5000    0.5000    1.0000

>> [v,d]=eig(A)

v =

   0.8602 + 0.0000i   0.8602 + 0.0000i   0.8602 + 0.0000i
   0.4136 + 0.0000i  -0.2068 + 0.3582i  -0.2068 - 0.3582i
   0.2982 + 0.0000i  -0.1491 - 0.2583i  -0.1491 + 0.2583i


d =

   3.1356 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0678 + 0.6486i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0678 - 0.6486i
```

```
A =

    1.0000    2.0000    6.0000    1.5000
    0.5000    1.0000    4.0000    0.2500
    0.1667    0.2500    1.0000    0.1667
    0.6667    4.0000    6.0000    1.0000

>> [v,d]=eig(A)

v =

  -0.6762 + 0.0000i   0.9394 + 0.0000i   0.1899 + 0.5501i   0.1899 - 0.5501i
  -0.2794 + 0.0000i   0.1152 + 0.0000i   0.0845 - 0.2718i   0.0845 + 0.2718i
  -0.0936 + 0.0000i  -0.1292 + 0.0000i   0.0555 + 0.0194i   0.0555 - 0.0194i
  -0.6752 + 0.0000i  -0.2958 + 0.0000i  -0.7595 + 0.0000i  -0.7595 + 0.0000i


d =

   4.1541 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0527 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0507 + 0.7950i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0507 - 0.7950i
```

```
A =

    1.0000    3.0000    6.0000    4.0000
    0.3330    1.0000    2.0000    0.3333
    0.1667    0.5000    1.0000    0.5000
    0.2500    3.0000    2.0000    1.0000

>> [v,d]=eig(A)

v =

   0.8992 + 0.0000i  -0.9821 + 0.0000i  -0.8557 + 0.0000i  -0.8557 + 0.0000i
   0.2151 + 0.0000i  -0.0409 + 0.0000i   0.1558 + 0.1908i   0.1558 - 0.1908i
   0.1358 + 0.0000i   0.1839 + 0.0000i  -0.0691 + 0.0319i  -0.0691 - 0.0319i
   0.3560 + 0.0000i   0.0004 + 0.0000i   0.2228 - 0.3893i   0.2228 + 0.3893i


d =

   4.2068 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0002 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.1033 + 0.9275i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.1033 - 0.9275i
```

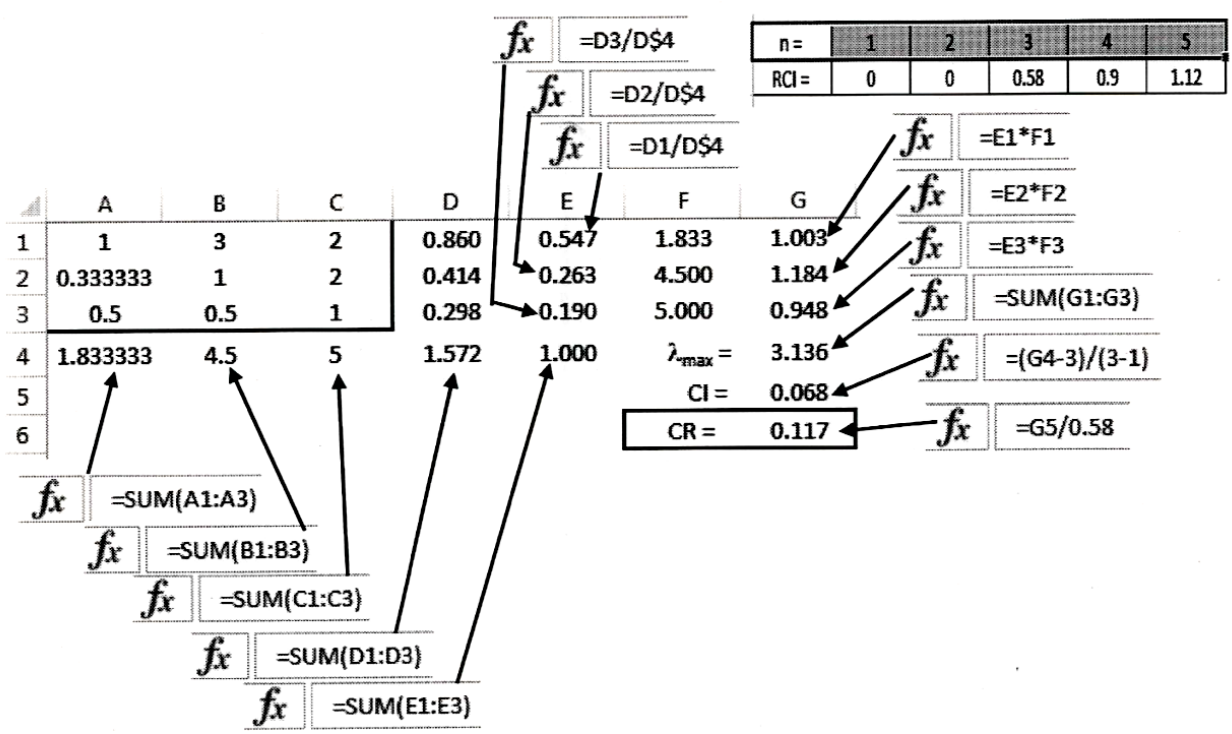
```
A =

    1.0000    3.0000    4.0000    3.0000
    0.3333    1.0000    0.5000    0.2500
    0.2500    2.0000    1.0000    0.1667
    0.3333    4.0000    6.0000    1.0000

>> [v,d]=eig(A)

v =

   0.8070 + 0.0000i  -0.8239 + 0.0000i  -0.8239 + 0.0000i  -0.8576 + 0.0000i
   0.1439 + 0.0000i   0.0086 + 0.1296i   0.0086 - 0.1296i   0.1956 + 0.0000i
   0.1702 + 0.0000i   0.1440 + 0.0211i   0.1440 - 0.0211i  -0.1829 + 0.0000i
   0.5469 + 0.0000i   0.0801 - 0.5261i   0.0801 + 0.5261i   0.4391 + 0.0000i


d =

   4.4115 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0220 + 1.3411i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0220 - 1.3411i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.3675 + 0.0000i
```

APPENDIX B

Pairwise Comparison Consistency Ratio

Note: Inconsistencies may be encountered when performing pairwise comparisons.

Consider the pairwise matrix in the bridge example above. Each element is the result from the decision tables. Each element must meet the criteria aij = aik x akj if all the comparisons are exactly consistent. . In actuality, the comparisons are frequently inconsistent. The elements in the pairwise matrix are considered to be satisfactorily consistent if the matrices consistency ratio (CR) is less than 10% (Saaty, 1980). The consistency index (CI) must be estimated to calculate the consistency ratio (CR). To estimate the consistency index (CI), the columns of the pairwise matrix are added and multiplied by the estimated eigen vector to yield the estimated maximum eigen value represented with the variable λmax, and CI = (λmax – n)/(n-1) where n is the size of the matrix. Then, CR = CI/ RCI where RCI is from the table below. The pairwise comparisons need to be revisited for CR values greater than 0.10. Both CR values above and below are shown below to illustrate the pairwise comparisons of each. For the 3x3 pairwise matrix from the AHP analysis comparing the variation type with respect to one another;



1	2	6	1.5	0.6762	0.392136	2.333333	0.914985
0.5	1	4	0.25	0.2794	0.162027	7.25	1.174698
0.166667	0.25	1	0.166667	0.0936	0.05428	17	0.922756
0.666667	4	6	1	0.6752	0.391556	2.916667	1.14204
2.333333	7.25	17	2.916667	1.7244		λ_{max} =	4.154479
						CI =	0.051493
						CR =	0.057214

1	3	6	4	0.8992	0.559866	1.75	0.979765
0.333333	1	2	0.333333	0.2151	0.133927	7.5	1.004452
0.166667	0.5	1	0.5	0.1358	0.084553	11	0.930079
0.25	3	2	1	0.356	0.221655	5.833333	1.292987
1.75	7.5	11	5.833333	1.6061		λ_{max} =	4.207283
						CI =	0.069094
						CR =	0.076771

1	3	4	3	0.807	0.483813	1.916667	0.927308
0.333333	1	0.5	0.25	0.1439	0.086271	10	0.86271
0.25	2	1	0.166667	0.1702	0.102038	11.5	1.173441
0.333333	4	6	1	0.5469	0.327878	4.416667	1.448126
1.916667	10	11.5	4.416667	1.668		λ_{max} =	4.411586
						CI =	0.137195
						CR =	0.152439

APPENDIX C

Monte Carlo Excel Calculations

Recall the pump illustration and our AHP analysis. The variation type and timing are determined by using a random number generator to assign values between 0 and 1 for columns A–H.

fx =ROUND(RAND(),3)

	A	B	C	D	E	F	G	H
9	Variation 1		Variation 2		Variation 3		Variation 4	
10	Type	Time	Type	Time	[redacted]			
11	0.15	0.29	0	0.52	0.61	0.21	0.69	0.65

For the type of variation, each random number is

	A	B	C	D	E	F	G	H
9	Variation 1		Variation 2		Variation 3		Variation 4	
10	Type	Time	Type	Time	[redacted]			
11	0.64	0.51	0.89	0.6	0.71	0.98	0.61	0.83

fx =IF($G11<=0.8,1,0)

fx =IF($A11>0.5,IF($A11<=0.9,1,0),0)

fx =IF($E11<=0.8,1,0)

fx =IF($C11>0.5,IF($C11<=0.9,1,0),0)

fx =IF($C11<=0.8,1,0)

fx =IF($A11>0.7,1,0)

fx =IF($A11<=0.8,1,0)

fx =IF($C11>0.7,1,0)

	I	J	K	L	M	N	O	P	Q	R	S	T	U
9			[redacted]				[redacted]				[redacted]		
10	Contract				Quality				[illegible]				
11	1	1	1	1	0	0	1	1	0	0	0	0	6
12	1	0	1	1	0	0	0	1	0	1	0	0	5
13	1	1	1	1	1	0	0	1	0	0	0	1	7

Recall the Gantt chart for the bridge illustration (figure 3.4) copied below for convenience. The timing of the variation is determined by the random number in columns B, D, F, and H. A binomial value is assigned to the impacted events (columns V-AK) to be used as a multiplier for calculating the impact value. All variations occur at a time greater than zero, so e_1 is assigned a value of 1 because every variation will impact at least some of the first event. The other equations shown below assign the binomial a value of 0 if the variation occurs before the event and a value of 1 if the variation occurs during or after the event.

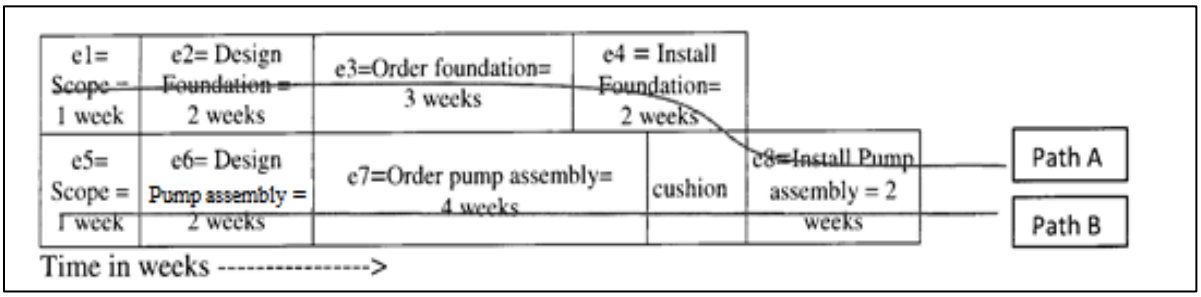


Figure 3.4 Two Parallel Coupled Sequential Events

Recall from the AHP analysis, T_1 = .5472, T2 = .2631 , T3 = .1897

and $[R]_1 = \begin{pmatrix} .392 \\ .162 \\ .054 \\ .392 \end{pmatrix}_1$ $[R]_2 = \begin{pmatrix} .560 \\ .134 \\ .084 \\ .222 \end{pmatrix}_2$ and $[R]_3 = \begin{pmatrix} .484 \\ .086 \\ .102 \\ .320 \end{pmatrix}_3$.

Also recall that the pump illustration uses T_1, T_2, and T_3 for the first as well as the second sequence of events. It also uses $[R]_1$, $[R]_2$, and $[R]_3$ for e_1 through e_4 as well as e_5 through e_6, and $[e_0]$ is given below for the two coupled sequence of events where e_4 must be completed before e_8.

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 3 \\ 6 \\ 0 \\ 3 \\ 5 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \\ 1 \\ 5 \\ 9 \\ 11 \end{bmatrix}$$

The Excel spreadsheet below is equations 2, 3, and 4 from page 30 copied for convenience. The calculation for Δe_1 is shown in cell AL11 below; similar calculations are made for Δe_2 through Δe_8.

$$\Delta = \sum_{k=1}^{K} \{ F_k \bullet M_k \} \circ [e] \qquad \underset{\sim}{F}_k = \frac{\mu_k}{\sum_{i=1}^{K} \mu_i T_i} T_k \qquad M_k = \frac{1}{\sum_{p=1}^{P} \rho_p^k R_p^k} [\rho]_k \circ [R]_k$$

Recall, $\underset{\sim}{F}_k$ is a multiplier that accounts for the variation type and M_k is the component that captures the different influence variation has on each event within each type of variation.

	A	B	C	D	E	F	G	H
9	Variation 1		Variation 2		Variation 3		Variation 4	
10	Type	Time	Type	Time				
11	0.64	0.51	0.89	0.6	0.71	0.98	0.61	0.83

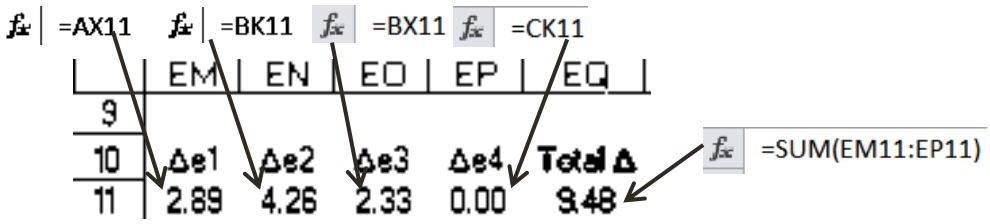
	I	J	K	L	M	N	O	P	Q	R	S	T	U
9													
10	Contract				Quality				Uncontrollable				
11	1	1	1	1	0	0	1	1	0	0	0	0	6
12	1	0	1	1	0	0	0	1	0	1	0	0	5

f_x =(0.5472/($I11*0.5472+$M11*0.263+$Q11*0.19))*(0.392/($V11*0.392+$W11*0.162+$X11*0.054+$Y11*0.392))*$I11*IF(($B11*8)<=1,($B11*8),1)

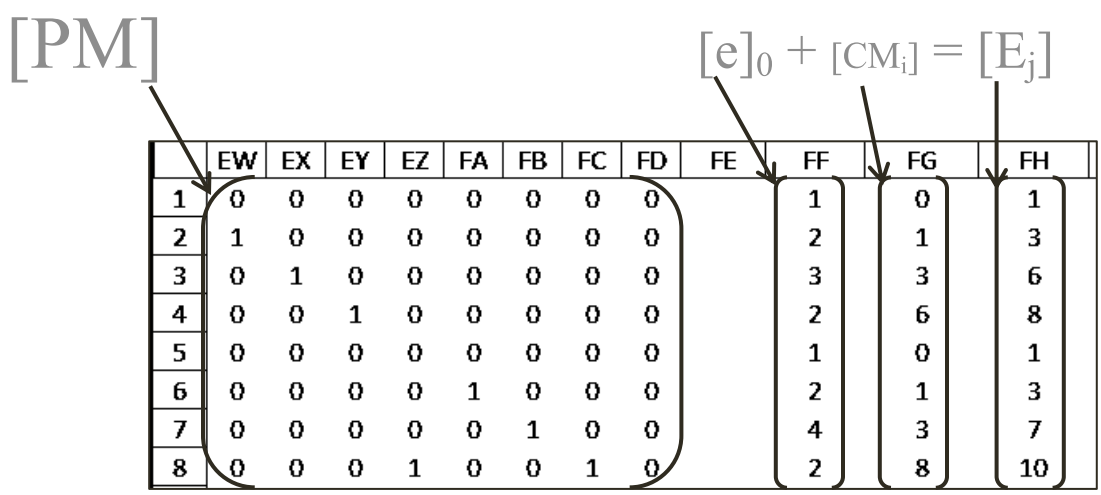
	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK
9																
10	e1	e2	e3	e4	e1	e2	e3	e4	e1	e2	e3	e4	e1	e2	e3	e4
11	1	1	0	0	1	1	1	0	1	1	0	0	1	1	1	0
12	1	0	0	0	1	1	0	0	1	0	0	0	1	1	0	0

	AL	AM	AN	AO	AP	AQ	AR	AS	AT	AU	AV	AW	AX
8													
9						Δe1							Δe1
10		Contract				Quality				Uncontrollable			Sum
11	0.7	0.6	0.5	0.4	0	0	0.3	0.3	0	0	0	0	2.89
12	0.8	0	0.6	0.5	0	0	0	0.3	0	0.9	0	0	3.10
13	0.5	0.7	0.6	0.4	0.3	0	0	0.2	0	0	0	0.2	2.90

The summation of each event is copied to columns EM through EP for convenience and summed in column EQ. Similar conventions are used for Δe_5 through Δe_8.

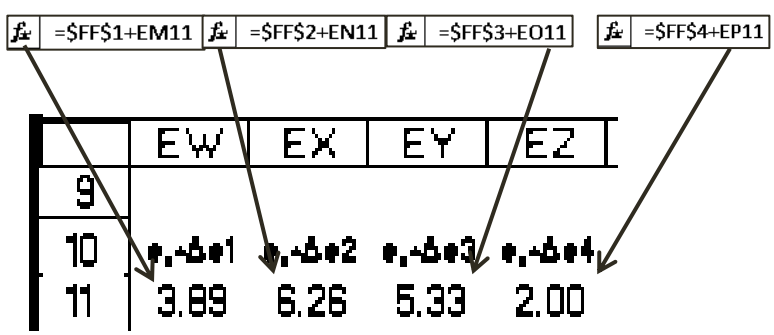


The precedence matrix [PM], initial event matrix $[e_0]$, cumulative matrix [CM] and the resulting final event completion time $[E]_j$ are in cells shown below.

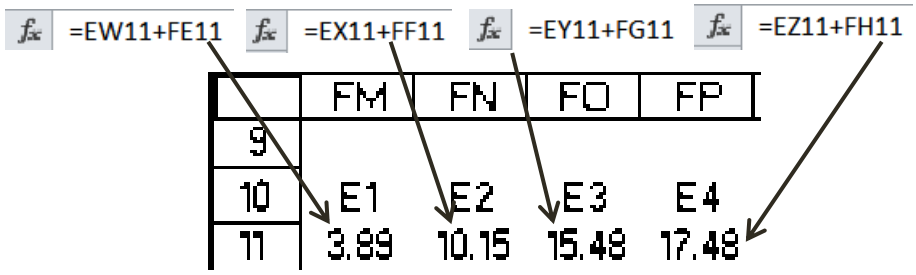


	EW	EX	EY	EZ	FA	FB	FC	FD	FE	FF	FG	FH
1	0	0	0	0	0	0	0	0		1	0	1
2	1	0	0	0	0	0	0	0		2	1	3
3	0	1	0	0	0	0	0	0		3	3	6
4	0	0	1	0	0	0	0	0		2	6	8
5	0	0	0	0	0	0	0	0		1	0	1
6	0	0	0	0	1	0	0	0		2	1	3
7	0	0	0	0	0	1	0	0		4	3	7
8	0	0	0	1	0	0	1	0		2	8	10

The new event matrix $[e]_1$ is calculated.

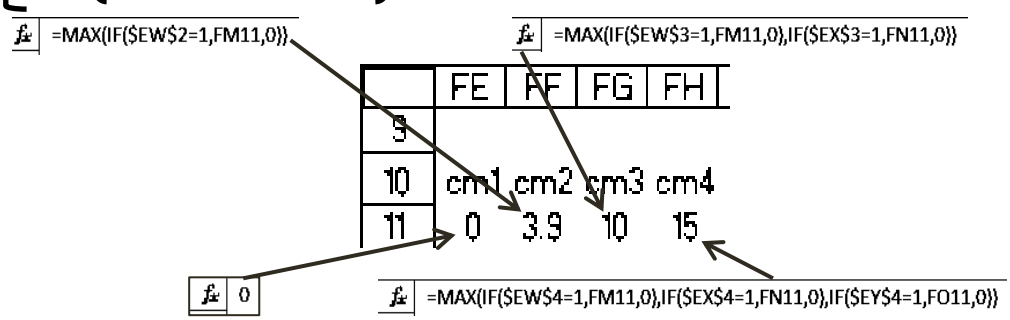


The matrix $[E]_1$ is updated to account for the variation. Excel avoids the circular reference error because the $[CM_i]$ matrix fills its column with the $[E_j]$ element where j is the row above the corresponding element CM_i.



[CM] is calculated from the formula below.

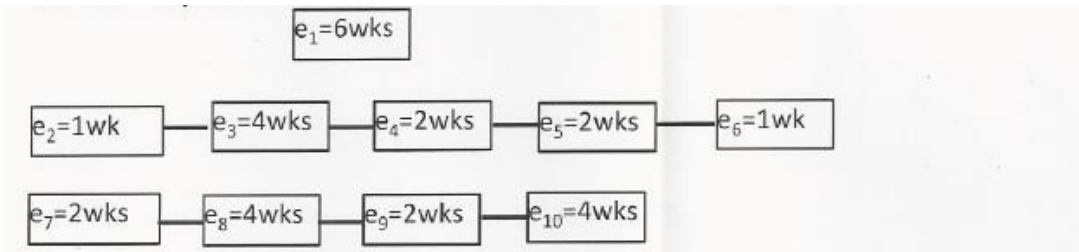
$$[CMi] = \begin{cases} 0 \text{ if } i = 1 \\ \text{Max}\left\{ \begin{array}{c} \text{IF}(PMij = 1, Ej, 0) \\ j=1,2,3\ldots.i-1 \end{array} \right\} \text{ if } i \neq 1 \end{cases} \qquad \text{Eq. 3.2}$$



APPENDIX D

Parallel Sequences with Constraints Located in the Upper Triangle of the Precedence Matrix

Below are three parallel uncoupled sequences of events.

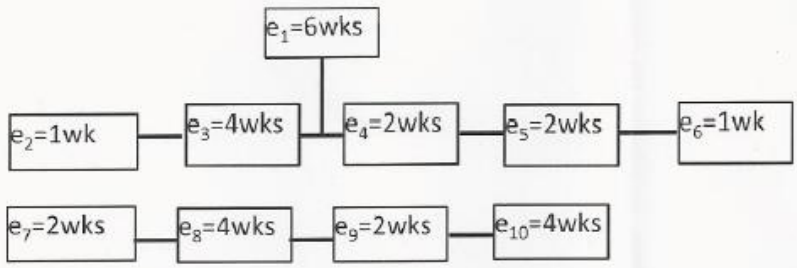


Below, [PM] is for the events shown above. Notice they are uncoupled.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	[e]	+	[CM]	=	[E]
e_1	0	0	0	0	0	0	0	0	0	1	6		0		6
e_2	0	0	0	0	1	0	0	0	0	0	1		0		1
e_3	0	1	0	0	0	0	0	0	0	0	4		1		5
e_4	0	0	1	0	0	0	0	0	0	0	2		5		7
e_5	0	0	0	1	0	0	0	1	0	0	2		7		9
e_6	0	0	0	0	1	0	0	0	0	0	1		9		10
e_7	0	0	0	0	0	0	0	0	0	0	2		0		2
e_8	0	0	0	0	0	0	1	0	0	0	4		2		6
e_9	0	0	0	0	0	0	0	1	0	0	2		6		8
e_{10}	0	0	0	0	0	0	0	0	1	0	4		8		12

The largest element in the column matrix [E] is 12, so the completion for the project above is 12 weeks.

Next, consider constraining the parallel events by requiring e1 to be completed before e4. Note the 1 in the precedence matrix located in row e4, column e1.

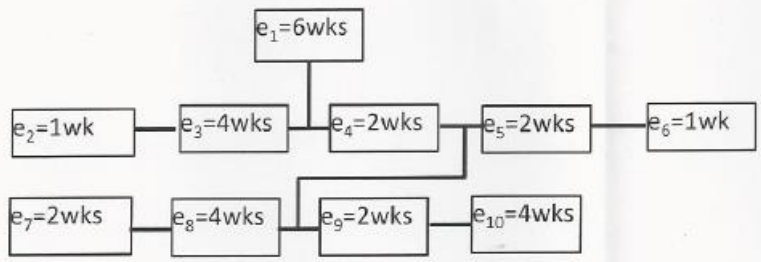


Below, [PM] is for the events shown above. Notice e1 needs to be completed before e4 can begin.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	[e]	+	[CM]	=	[E]
e_1	0	0	0	0	0	0	0	0	0	0	6		0		6
e_2	0	0	0	0	0	0	0	0	0	0	1		0		1
e_3	0	1	0	0	0	0	0	0	0	0	4		1		5
e_4	1	0	1	0	0	0	0	0	0	0	2		6		8
e_5	0	0	0	1	0	0	0	0	0	0	2		8		10
e_6	0	0	0	0	1	0	0	0	0	0	1		10		11
e_7	0	0	0	0	0	0	0	0	0	0	2		0		2
e_8	0	0	0	0	0	0	1	0	0	0	4		2		6
e_9	0	0	0	0	0	0	0	1	0	0	2		6		8
e_{10}	0	0	0	0	0	0	0	0	1	0	4		8		12

The largest element in the column matrix [E] is 12, so the completion for the project above is 12 weeks, but the second sequence of events requires 11 weeks for completion instead of the 10 weeks as shown in the uncoupled scenario.

Next, consider adding another constraint to the parallel events by not only requiring e1 to be completed before e4, but also e4 must be completed before e9. Note the 1 in the precedence matrix located in row e4, column e1 and in row e9 column e4.



Below, [PM] is for the events shown above. Notice e1 needs to be completed before e4 can begin, and e4 must be completed before e9 can begin.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	[e]	+	[CM]	=	[E]
e_1	0	0	0	0	0	0	0	0	0	0	6		0		6
e_2	0	0	0	0	0	0	0	0	0	0	1		0		1
e_3	0	1	0	0	0	0	0	0	0	0	4		1		5
e_4	1	0	1	0	0	0	0	0	0	0	2		6		8
e_5	0	0	0	1	0	0	0	0	0	0	2		8		10
e_6	0	0	0	0	1	0	0	0	0	0	1		10		11
e_7	0	0	0	0	0	0	0	0	0	0	2		0		2
e_8	0	0	0	0	0	0	1	0	0	0	4		2		6
e_9	0	0	0	1	0	0	0	1	0	0	2		8		10
e_{10}	0	0	0	0	0	0	0	0	1	0	4		10		14

[PM]

The largest element in the column matrix [E] is 14, so the completion for the project above is 14 weeks.

In the examples above, the constraints fell in the lower half of the precedence matrix because the sequence of events are ordered from left to right, and the constraints are arranged in order from top to bottom. If a constraint is arranged from bottom to top, the 1 falls in the upper triangle of the precedence matrix, and the formula results in a circular error reference. To avoid this error, the following convention is used.

Equation

$$[e] + [CM] = [E] \qquad \text{Eq. 3.1}$$

can be modified as

$$[e] + MAX\{[CM]_L, [CM]_U\} = [E] \qquad \text{Eq. D.1}$$

where $[CM]_L =$

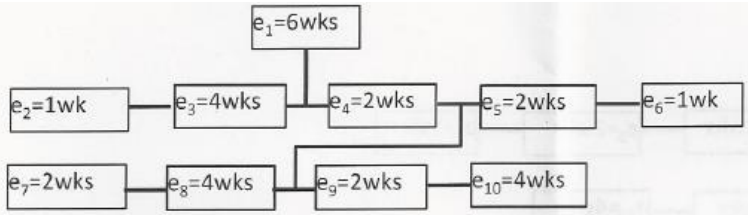
$$[CM]_L = \begin{cases} 0 \text{ if } i = 1 \\ \text{Max}\left\{ \begin{matrix} IF(PMij = 1, Ej, 0) \\ j = 1,2,3\ldots i\text{-}1 \end{matrix} \right\} \text{ if } i \neq 1 \end{cases} \qquad \text{Eq. 3.2}$$

and $[CM]_U =$

$$[CM]_U = \begin{cases} 0 \text{ if } i = q \\ \text{Max}\left\{ \begin{matrix} IF(PMij = 1, Ej, 0) \\ j = i\text{+}1, i\text{+}2..q \end{matrix} \right\} \text{ if } i \neq q \end{cases} \qquad \text{Eq. D.2}$$

where q is the size of the precedence matrix.

This convention yields the same results as equation 3.2 in the uncoupled and one constraint situation above, but if e8 is required to be completed before starting e5, the 1 falls in the upper half of the precedence matrix, and this convention yields the same results as equation 3.2 but considers the 8 and 6 in row e5 using the larger of the two to calculate [E].



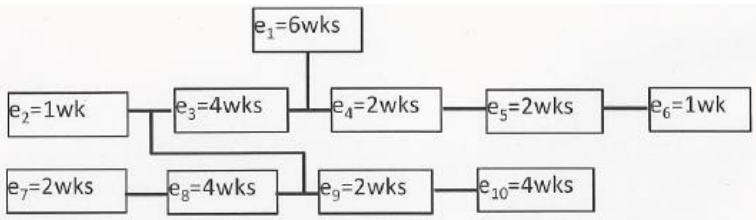
Below, [PM] is for the events shown above. Notice e1 needs to be completed before e4 can begin, and e8 must be completed before e5 can begin.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	[e]	+ MAX ($[CM]_L$,	$[CM]_U$) =	[E]
e_1	0	0	0	0	0	0	0	0	0	0	6	0	0	6
e_2	0	0	0	0	0	0	0	0	0	0	1	0	0	1
e_3	0	1	0	0	0	0	0	0	0	0	4	1	0	5
e_4	1	0	1	0	0	0	0	0	0	0	2	6	0	8
e_5	0	0	0	1	0	0	0	1	0	0	2	8	6	10
e_6	0	0	0	0	1	0	0	0	0	0	1	10	0	11
e_7	0	0	0	0	0	0	0	0	0	0	2	0	0	2
e_8	0	0	0	0	0	0	1	0	0	0	4	2	0	6
e_9	0	0	0	0	0	0	0	1	0	0	2	6	0	8
e_{10}	0	0	0	0	0	0	0	0	1	0	4	8	0	12

[PM]

On the next page is a situation where a 1 located in the upper half of the precedence matrix is the determining factor for final completion time.

Consider constraining the events by requiring e1 to be completed before e4, but if e8 is required to be completed before starting e3, the 1 falls in the upper half of the precedence matrix, and this convention yields a different result than equation 3.2 because it considers the 1 and 6 in row e3 using the larger of the two to calculate [E].



Below, [PM] is for the events shown above. Notice e1 needs to be completed before e4 can begin, and e8 must be completed before e3 can begin.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	[e]	+ MAX ($[CM]_L$,	$[CM]_U$) =	[E]
e_1	0	0	0	0	0	0	0	0	0	0	6	0	0	6
e_2	0	0	0	0	0	0	0	0	0	0	1	0	0	1
e_3	0	1	0	0	0	0	0	1	0	0	4	1	6	10
e_4	1	0	1	0	0	0	0	0	0	0	2	10	0	12
e_5	0	0	0	1	0	0	0	0	0	0	2	12	0	14
e_6	0	0	0	0	1	0	0	0	0	0	1	14	0	15
e_7	0	0	0	0	0	0	0	0	0	0	2	0	0	2
e_8	0	0	0	0	0	0	1	0	0	0	4	2	0	6
e_9	0	0	0	0	0	0	0	1	0	0	2	6	0	8
e_{10}	0	0	0	0	0	0	0	0	1	0	4	8	0	12

The largest element in the column matrix [E] is 15, so the completion for the project above is 15 weeks.

VITA

Thomas V. Hammock has served 20 years with various manufacturing companies as Plant Manager, Director of Manufacturing, and Maintenance Supervisor. He presently works as a test engineer at Newport News Shipbuilding. He received his M.S. in Mechanical Engineering at Virginia Tech. and an M.B.A. from University of Richmond. Thomas performed his PhD studies at Old Dominion University, Batten College of Engineering & Technology, 1105 Engineering Systems Building, Norfolk, VA 23529.